

LINEAR INEQUALITIES

1. If $3^x + 2^{2x} \geq 5^x$, then the solution set for x is
 a) $(-\infty, 2]$ b) $[2, \infty)$ c) $[0, 2]$ d) $\{2\}$
2. $x^2 - 3|x| + 2 < 0$, then x belongs to
 a) $(1, 2)$ b) $(-2, -1)$ c) $(-2, -1) \cup (1, 2)$ d) $(-3, 5)$
3. Solution of $2^x + 2^{|x|} \geq 2\sqrt{2}$ is
 a) $(-\infty, \log_2(\sqrt{2} + 1))$ b) $(0, 8)$
 c) $(\frac{1}{2}, \log_2(\sqrt{2} - 1))$ d) $(-\infty, \log_2(\sqrt{2} - 1)) \cup [\frac{1}{2}, \infty)$
4. If x_1, x_2, \dots, x_n are real numbers, then the largest value of the expression $\sin x_1 \cos x_2 + \sin x_2 \cos x_3 + \dots + \sin x_n \cos x_1$ is
 a) n b) $\frac{n}{2}$ c) $\frac{n}{4}$ d) $\sqrt{n^2 - 1}$
5. If $a < b$, then the solution $x^2 + (a + b)x + ab < 0$ is given by
 a) (a, b) b) $(-\infty, a) \cup (b, \infty)$ c) $(-b, -a)$ d) $(-\infty, -b) \cup (-a, \infty)$
6. If $\log(x^3 + y^3) - \log_{10}(x^2 + y^2 - xy) \leq 2$, then the maximum value of xy , for all $x \geq 0, y \geq 0$ is
 a) 2500 b) 3000 c) 1200 d) 3500
7. If $3^{x/2} + 2^x > 25$, then the solution set is
 a) R b) $(2, \infty)$ c) $(4, \infty)$ d) None of these
8. If $ab = 4(a, b \in R^+)$, then
 a) $a + b \leq 4$ b) $a + b = 4$ c) $a + b \geq 4$ d) None of these
9. Let $P_n(x) = 1 + 2x + 3x^2 + \dots + (n + 1)x^n$ be a polynomial such that n is even. Then, the number of real roots of $P_n(x)$, is
 a) 0 b) n c) 1 d) None of these
10. $(x - 1)(x^2 - 5x + 7) < (x - 1)$, then x belongs to
 a) $(1, 2) \cup (3, \infty)$ b) $(2, 3)$ c) $(-\infty, 1) \cup (2, 3)$ d) None of these
11. If $x = \log_{2^2} 2 + \log_{2^3} 2^2 + \log_{2^4} 2^3 + \dots + \log_{2^{n+1}} 2^n$, then
 a) $x \geq \left(\frac{1}{n+1}\right)^{1/n}$ b) $x \geq n \left(\frac{1}{n+1}\right)^{1/n}$ c) $x \geq \left(\frac{n}{n+1}\right)^{1/n}$ d) None of these
12. The number of real solutions (x, y, z, t) of simultaneous equations
 $2y = \frac{11}{x} + x, 2z = \frac{11}{y} + y, 2t = \frac{11}{z} + z, 2x = \frac{11}{t} + t$, is
 a) 0 b) 1 c) 2 d) 4
13. The solution set contained in R of the inequation $3^x + 3^{1-x} - 4 < 0$, is
 a) $(1, 3)$ b) $(0, 1)$ c) $(1, 2)$ d) $(0, 2)$
14. The range of ab if $|a| \leq 1$ and $a + b = 1, (a, b \in R)$, is
 a) $[0, 1/4]$ b) $[-2, 1/4]$ c) $[1/4, 2]$ d) $[0, 2]$
15. If $\sqrt{9x^2 + 6x + 1} < (2 - x)$, then
 a) $x \in \left(-\frac{3}{2}, \frac{1}{4}\right)$ b) $x \in \left(-\frac{3}{2}, \frac{1}{4}\right)$ c) $x \in \left[-\frac{3}{2}, \frac{1}{4}\right]$ d) $x < \frac{1}{4}$
16. If $5^x + (2\sqrt{3})^{2x} \geq 13^x$, then the solution set for x is
 a) $[2, \infty)$ b) $\{2\}$ c) $(-\infty, 2]$ d) $[0, 2]$

17. Solution set of inequality $\log_e \frac{x-2}{x-3}$ is
 a) $(2, \infty)$ b) $(-\infty, 2)$ c) $(-\infty, \infty)$ d) $(3, \infty)$
18. If $3 < 3t - 18 \leq 18$, then which one of the following is true?
 a) $15 \leq 2t + 1 \leq 20$ b) $8 \leq t < 12$ c) $8 \leq t + 1 \leq 13$ d) $21 \leq 3t \leq 24$
19. Let $f(x) = ax^2 + bx + c$ and $f(-1) < 1, f(1) > -1, f(3) < -4$ and $a \neq 0$, then
 a) $a > 0$ b) $a < 0$
 c) Sign of a cannot be determined d) None of the above
20. The set of admissible values of x such that $\frac{2x+3}{2x-9} < 0$ is
 a) $(-\infty, -\frac{3}{2}) \cup (\frac{9}{2}, \infty)$ b) $(-\infty, 0) \cup (\frac{9}{2}, \infty)$ c) $(-\frac{3}{2}, 0)$ d) $(-\frac{3}{2}, \frac{9}{2})$
21. The number of irrational solutions of the equation $\sqrt{x^2 + \sqrt{x^2 + 11}} + \sqrt{x^2 - \sqrt{x^2 + 11}} = 4$, is
 a) 0 b) 2 c) 4 d) 11
22. The number of solutions of the equation $\log_{x-3}(x^3 - 3x^2 - 4x + 8) = 3$, is
 a) 1 b) 2 c) 3 d) 4
23. The number of real solutions of the equation $\log_{0.5} x = |x|$, is
 a) 1 b) 0 c) 2 d) None of these
24. The number of complex roots of the equation $x^4 - 4x - 1 = 0$, is
 a) 3 b) 2 c) 1 d) 0
25. If $\sin^x \alpha + \cos^x \alpha \geq 1, 0 < \alpha < \frac{\pi}{2}$, then
 a) $x \in (2, \infty)$ b) $x \in (-\infty, 2]$ c) $x \in [-1, 1]$ d) None of these
26. Consider the following statements:
 1. $\frac{x}{1+x^2} < \tan^{-1} x < x; x > 0$
 2. If $0 \leq x < \frac{\pi}{2}, \sin x + \tan x - 3x \geq 0$
 Which of these is/are correct?
 a) Only (1) b) Only (2) c) (1) and (2) d) None of these
27. The number of solutions of the equation $2 \cos(e^x) = 3^x + 3^{-x}$, is
 a) 0 b) 1 c) 2 d) None of these
28. The number of real solutions of the equation $1 - x = [\cos x]$, is
 a) 1 b) 2 c) 3 d) None of these
29. Non- negative real numbers such that $a_1 + a_2 + \dots + a_n = p$ and $q = \sum_{i < j} a_i a_j$, then
 a) $q \leq \frac{1}{2} p^2$ b) $q > \frac{1}{4} p^2$ c) $q < \frac{p}{2}$ d) $q > \frac{p^2}{2}$
30. If $(\sin a)^x + (\cos a)^x \geq 1, 0 < a < \frac{\pi}{2}$, then
 a) $x \in [2, \infty)$ b) $x \in (-\infty, 2]$ c) $x \in [-1, 1]$ d) None of these
31. If $x^2 + 2ax + 10 - 3a > 0$ for all $x \in R$, then
 a) $-5 < a < 2$ b) $a < -5$ c) $a > 5$ d) $2 < a < 5$
32. The least integer satisfying $49.4 - \left(\frac{27-x}{10}\right) < 47.4 - \left(\frac{27-9x}{10}\right)$, is
 a) 2 b) 3 c) 4 d) None of these
33. For positive real number a, b, c which one of the following holds?
 a) $a^2 + b^2 + c^2 \geq bc + ca + ab$ b) $(b+c)(c+a)(a+b) \leq 8abc$
 c) $\frac{a}{b} + \frac{b}{c} + \frac{c}{a} \leq 3$ d) $a^3 + b^3 + c^3 \leq 3abc$
34. The least perimeter of a cyclic quadrilateral of given area A square units is
 a) \sqrt{A} b) $2\sqrt{A}$ c) $3\sqrt{A}$ d) $4\sqrt{A}$
35. The number of solutions of $[\sin x + \cos x] = 3 + [-\sin x] + [-\cos x]$ in the interval $[0, 2\pi]$ is (where $[\]$ denotes the greatest integer function)
 a) 0 b) 4 c) Infinite d) 1

36. The number of solutions of $3^{|x|} = |2 - |x||$ is
 a) 0 b) 2 c) 4 d) Infinite
37. If C is an obtuse angle in triangle, then
 a) $\tan A \tan B < 1$ b) $\tan A \tan B > 1$ c) $\tan A \tan B = 1$ d) None of these
38. If x, y, z are three real numbers such that $x + y + z = 4$ and $x^2 + y^2 + z^2 = 6$, then the exhaustive set of values of x , is
 a) $[2/3, 2]$ b) $[0, 2/3]$ c) $[0, 2]$ d) $[-1/3, 2/3]$
39. The number of roots of the equation $[\sin^{-1} x] = x - [x]$, is
 a) 0 b) 1 c) 2 d) None of these
40. If $3^{x/2} + 2^x > 25$, then
 a) $x \in [4, \infty)$ b) $(4, \infty)$ c) $x \in (-\infty, 4]$ d) $x \in [0, 4]$
41. The number of real solutions of the equation $27^{1/x} + 12^{1/x} = 2 \cdot 8^{1/x}$, is
 a) 1 b) 2 c) 0 d) Infinite
42. If roots of the equation $x^4 - 8x^3 + bx^2 + cx + 16 = 0$ are positive then
 a) $b = 8 = c$ b) $b = -24, c = -32$ c) $b = 24, c = -32$ d) $b = 24, c = 32$
43. If $3 < |x| < 6$, then x belongs to
 a) $(-6, -3) \cup (3, 6)$ b) $(-6, 6)$ c) $(-3, -3) \cup (3, 6)$ d) None of these
44. If a, b are distinct positive real numbers, then which one of the following is true?
 a) $a^4 + b^4 > a^3b + ab^3$ b) $a^4 + b^4 < a^3b + ab^3$ c) $a^3 + b^3 < a^2b + ab^2$ d) None of these
45. The solution of the inequality $4^{-x+0.5} - 7 \cdot 2^{-x} < 4, x \in R$ is
 a) $(-2, \infty)$ b) $(2, \infty)$ c) $(2, \frac{7}{2})$ d) None of these
46. Suppose a, b and c are real numbers such that $\frac{a}{b} > 1$ and $\frac{a}{c} < 0$. Which one of the following is true?
 a) $a + b - c > 0$ b) $a > b$ c) $(a - c)(b - c) > 0$ d) $a + b + c > 0$
47. If a, b, c are positive real numbers such that $a + b + c = p$ then, which of the following is true?
 a) $(p - a)(p - b)(p - c) \geq \frac{1}{27}p^3$
 b) $(p - a)(p - b)(p - c) \geq 8abc$
 c) $\frac{bc}{a} + \frac{ca}{b} + \frac{ab}{c} \geq p$
 d) None of these
48. The number of solutions of the equation $\frac{(1+e^{x^2})\sqrt{1+x^2}}{\sqrt{1+x^4-x^2}} = 1 + \cos x$, is
 a) 1 b) 2 c) 3 d) 4
49. Let n be an odd integer such that the polynomial $P_n(x) = 1 + 2x + 3x^2 + \dots + (n + 1)x^n$ has exactly one real root. This real root α satisfies
 a) $-1 < \alpha < 0$ b) $0 < \alpha < 1$ c) $0 \leq \alpha \leq 1$ d) $-1 \leq \alpha \leq 0$
50. Let a, b be integers and $f(x)$ be a polynomial with integer coefficients such that $f(b) - f(a) = 1$. Then, the value of $b - a$, is
 a) 1 b) -1 c) 1, -1 d) None of these
51. Let $y = \sqrt{\frac{(x+1)(x-3)}{(x-2)}}$, then all real values of x for which y takes real values, are
 a) $-1 \leq x < 2$ or $x \geq 3$ b) $-1 \leq x < 3$ or $x > 2$ c) $1 \leq x < 2$ or $x \geq 3$ d) None of these
52. If $a, b, c > 0$ and if $abc = 1$, then the value of $a + b + c + ab + bc + ca$ lies in the interval
 a) $(\infty, -6)$ b) $(-6, 0)$ c) $(0, 6)$ d) $(6, \infty)$
53. The number of real roots of the equation $(\sin 2^x)(\cos 2^x) = \frac{2^x + 2^{-x}}{2}$, is
 a) 1 b) 2 c) 3 d) None of these
54. The largest interval for which $x^{12} - x^9 + x^4 - x + 1 > 0$ is
 a) $-4 < x \leq 0$ b) $0 < x < 1$ c) $-100 < x < 100$ d) $0 < x < \infty$
55. The number of negative real roots of $x^4 - 4x - 1 = 0$, is

- a) 3 b) 2 c) 1 d) 0
56. If $0 < x < \frac{\pi}{2}$, then minimum value of $\frac{\cos^3 x}{\sin x} + \frac{\sin^3 x}{\cos x}$ is
a) $\sqrt{3}$ b) $\frac{1}{2}$ c) $\frac{1}{3}$ d) 1
57. The number of solutions of $\sqrt{3x^2 + 6x + 7} + \sqrt{5x^2 + 10x + 14} = 4 - 2x - x^2$, is
a) 1 b) 2 c) 3 d) 4
58. The solution set of $||x|-1| < |1-x|$, $x \in R$ is
a) $(-1, 1)$ b) $(0, \infty)$ c) $(-1, \infty)$ d) None of these
59. The minimum value of $f(x) = |3-x| + 7$ is
a) 0 b) 6 c) 7 d) 8
60. The solution set of the inequation $\frac{x+11}{x-3} > 0$ is
a) $(-\infty, 11) \cup (3, \infty)$ b) $(-\infty, -10) \cup (2, \infty)$ c) $(-100, -11) \cup (1, \infty)$ d) $(-5, 0) \cup (3, 7)$
61. Solution of $2x - 1 = |x + 7|$ is
a) -2 b) 8 c) -2, 8 d) None of these
62. The number of positive real roots of $x^4 - 4x - 1 = 0$, is
a) 3 b) 2 c) 1 d) 0
63. The solution set of the inequality $\log_{\sin(\frac{\pi}{3})}(x^2 - 3x + 2) \geq 2$ is
a) $(\frac{1}{2}, 2)$ b) $(1, \frac{5}{2})$ c) $[\frac{1}{2}, 1) \cup (2, \frac{5}{2}]$ d) None of these
64. If a, b, c are sides of triangle, then $\frac{(a+b+c)^2}{(ab+bc+ca)}$ always belongs to
a) $[1, 2]$ b) $[2, 3]$ c) $[3, 4]$ d) $[4, 5]$
65. $(x-1)(x^2 - 5x + 7) < (x-1)$, then x belongs to
a) $(1, 2) \cup (3, \infty)$ b) $(-\infty, 1) \cup (2, 3)$ c) 2, 3 d) None of these
66. The set of values of x for which the inequalities $x^2 - 3x - 10 < 0$, $10x - x^2 - 16 > 0$ hold simultaneously, is
a) $(-2, 5)$ b) $(2, 8)$ c) $(-2, 8)$ d) $(2, 5)$
67. The solution of the inequation $\log_{1/3}(x^2 + x + 1) + 1 < 0$ is
a) $(-\infty, -2) \cup (1, \infty)$ b) $[-1, 2]$ c) $(-2, 1)$ d) $(-\infty, \infty)$
68. The number of values of a for which the system of equation $2^{|x|} + |x| = y + x^2 + a$ and $x^2 + y^2 = 1$ has only one solution where a, x, y are real, is
a) 1
b) 2
c) Finitely many but more than 2
d) Infinitely many
69. The solution set of the inequation $\log_{1/3}(x^2 + x + 1) + 1 > 0$, is
a) $(-\infty, -2) \cup (1, \infty)$ b) $[-1, 2]$ c) $(-2, 1)$ d) R
70. The set of all real numbers satisfying the inequation $x^{(\log_{10} x)^2 - 3(\log_{10} x) + 1} > 1000$, is
a) $(0, 1000)$ b) $(1000, \infty)$ c) $(0, 100)$ d) None of these
71. The set of all real x satisfying the inequality $\frac{3-|x|}{4-|x|} > 0$
a) $[-3, 3] \cup (-\infty, -4) \cup (4, \infty)$ b) $(-\infty, -4) \cup (4, \infty)$
c) $(-\infty, -3) \cup (4, \infty)$ d) $(-\infty, -3) \cup (3, \infty)$
72. For positive real number a, b, c which of the following holds?
a) $a + b + c > 3 \Rightarrow a^2 + b^2 + c^2 > 3$ b) $a^6 + b^6 \leq 12a^2 b^2 - 64$
c) $a + b + c = \alpha \Rightarrow \frac{1}{a} + \frac{1}{b} + \frac{1}{c} \leq \frac{9}{\alpha}$ d) None of the above
73. If a_1, a_2, a_3 be any positive real numbers, then which of the following statements is not true
a) $3a_1, a_2, a_3 \leq a_1^3 + a_2^3 + a_3^3$

- b) $\frac{a_1}{a_2} + \frac{a_2}{a_3} + \frac{a_3}{a_1} \geq 3$
- c) $(a_1 + a_2 + a_3) \left(\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3} \right) \geq 9$
- d) $(a_1 + a_2 + a_3) \left(\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3} \right)^3 \leq 27$
74. If $x^2 + \frac{1}{x^2} = A$ and $x - \frac{1}{x} = B$, where $x \in R$ and $B > 0$, then the minimum value of $\frac{A}{B}$ is
 a) $\sqrt{2}$ b) $2\sqrt{2}$ c) $\sqrt{2} + 2$ d) None of these
75. Let n be an odd positive integer. Then, the number of real roots of the polynomial $P_n(x) = 1 + 2x + 3x^2 + \dots + (n+1)x^n$, is
 a) 0 b) n c) 1 d) None of these
76. The number of positive integers satisfying the inequality $n + 1_{C_{n-2}} - n + 1_{C_{n-1}} \leq 50$ is
 a) 9 b) 8 c) 7 d) 6
77. For $\theta > \pi/3$, the value of $f(\theta) = \sec^2 \theta + \cos^2 \theta$ always lies in the interval
 a) $(0, 2)$ b) $[0, 1]$ c) $(1, 2)$ d) $[2, \infty)$
78. If the product of n positive numbers is n^n , then their sum is
 a) A positive integer b) Divisible by n c) Equal to $n + \frac{1}{n}$ d) Never less than n^2
79. $\log_2(x^2 - 3x + 18) < 4$, then x belongs to
 a) $(1, 2)$ b) $(2, 16)$ c) $(1, 16)$ d) None of these
80. If $[x]^2 = [x + 6]$, where $[x]$ = the greatest integer less than or equal to x , then x must be such that
 a) $x = 3, -2$ b) $x \in [-2, -1)$ c) $x \in [3, 4)$ d) $x \in [-2, -1) \cup [3, 4)$
81. If $a, b, c > 0$, the minimum value of $\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b}$ is
 a) 1 b) $\frac{3}{2}$ c) 2 d) $\frac{5}{2}$
82. The number of real solutions of the equation $e^{|x|} - |x| = 0$, is
 a) 0 b) 1 c) 2 d) None of these
83. If p, q, r are any real numbers, then
 a) $\max(p, q) = \max(p, q, r)$ b) $\min(p, q) = \frac{1}{2}(p + q - |p - q|)$
 c) $\max(p, q) < \min(p, q, r)$ d) $\max(p, q) = \frac{1}{2}(p + q - |p - q|)$
84. The number of real solutions of $1 + |e^x - 1| = e^x(e^x - 2)$, is
 a) 1 b) 2 c) 3 d) 4
85. The solution set of the inequation $\log_{\sin 2\pi/3}(x^2 - 3x + 2) \geq 2$, is
 a) $[1/2, 1)$ b) $(2, 5/2]$ c) $[1/2, 1) \cup (2, 5/2]$ d) $[1/2, 5/2]$
86. If $x^2 + 6x - 27 < 0$ and $x^2 - 3x - 4 < 0$, then
 a) $x > 3$ b) $x < 4$ c) $3 < x < 4$ d) $x = \frac{7}{2}$
87. If $x, y \in R$, then $\frac{1}{2}(x + y + |x - y|) = x$ holds iff
 a) $x > y$ b) $x < y$ c) $x = y$ d) $x \geq y$
88. The set of all x satisfying the inequality $\frac{4x-1}{3x+1} \geq 1$ is
 a) $\left(-\infty, -\frac{1}{3}\right) \cup \left[\frac{1}{4}, \infty\right)$ b) $\left(-\infty, -\frac{2}{3}\right) \cup \left[\frac{5}{4}, \infty\right)$ c) $\left(-\infty, -\frac{1}{3}\right) \cup [2, \infty)$ d) $\left(-\infty, -\frac{2}{3}\right) [4, \infty)$
89. The number of solutions of the inequality $E = 2^{1/\sin^2 \alpha_2} \cdot 3^{1/\sin^2 \alpha_3} \dots n^{1/\sin^2 \alpha_n} < n!$
 Where $\alpha_i \in (-\pi, 2\pi)$ for $i = 2, 3, \dots, n$ is
 a) 0 b) 2^{n-1} c) 3^{n-1} d) None of these
90. The equation $e^x = x(x+1)$, $x < 0$ has

- a) No real roots set
 b) Exactly one real root
 c) Two real roots
 d) Infinitely many real roots
91. Let $F(x)$ be a function defined by $F(x) = x - [x]$, $0 \neq x \in R$, where $[x]$ is the greatest integer less than or equal to x . Then, the number of solutions of $F(x) + F\left(\frac{1}{x}\right) = 1$
- a) 0 b) Infinite c) 1 d) 2
92. The set of all real numbers satisfying the inequation $2^x + 2^{|x|} \geq 2\sqrt{2}$, is
- a) $(1/2, \infty)$
 b) $(-\infty, \log_2(\sqrt{2} - 1))$
 c) $(-\infty, 1/2)$
 d) $[1/2, \infty) \cup (-\infty, \log_2(\sqrt{2} - 1))$
93. Solution of the inequality $\sin^4\left(\frac{x}{3}\right) + \cos^4\left(\frac{x}{3}\right) > \frac{1}{2}$, is given by
- a) R b) $\frac{3n\pi}{2} + \frac{3\pi}{4}$
 c) $R - \left\{\left(\frac{3n\pi}{2} + \frac{3\pi}{4}\right), n \in I\right\}$ d) None of these
94. If $(\log_5 x)^2 + (\log_5 x) < 2$, then x belongs to the interval
- a) $(1/25, 5)$ b) $(1/5, 1/\sqrt{5})$ c) $(1, \infty)$ d) None of these
95. The number of real roots of the equation $x^2 + x + 3 + 2 \sin x = 0$ in the interval $[-\pi, \pi]$, is
- a) 2 b) 4 c) 6 d) None of these
96. If $\log_{\sqrt{3}}(\sin x + 2\sqrt{2} \cos x) \geq 2$, $-2\pi \leq x \leq 2\pi$, then the number of solutions of x is
- a) 0 b) ∞ c) 3 d) 4
97. Solution of $x^{(\log_{10} x)^2 - 3 \log_{10} x + 1} > 1000$ for $x \in R$, is
- a) $(10, \infty)$ b) $(100, \infty)$ c) $(1000, \infty)$ d) $(1, \infty)$
98. The largest interval for which $x^{12} - x^9 + x^4 - x + 1 > 0$ is
- a) $-4 < x < 0$ b) $0 < x < 1$ c) $-100 < x < 100$ d) $-\infty < x < \infty$
99. The number of roots of the equation $\sin \pi x = |\log|x||$, is
- a) 2 b) 4 c) 5 d) 6
100. The equation $\sqrt{x+1} - \sqrt{x-1} = \sqrt{4x-1}$ has
- a) No solution
 b) One solution
 c) Two solutions
 d) More than two solutions
101. If $\log_3 x - \log_x 27 < 2$, then x belongs to the interval
- a) $(1/3, 27)$ b) $(1/27, 3)$ c) $(1/9, 9)$ d) None of these
102. The set of all solutions of the inequation $x^2 - 2x + 5 \leq 0$ in R is
- a) $R - (-\infty, -5)$ b) $R - (5, \infty)$ c) \emptyset d) $R - (-\infty, -4)$
103. $2^{\sin^2 x} + 2^{\cos^2 x}$ is
- a) ≤ 2 b) ≥ 2 c) ≤ 1 d) ≥ 1
104. If $a^2 + b^2 + c^2 = 1$, then $ab + bc + ca$ lies in the interval
- a) $\left[-\frac{1}{2}, 1\right]$ b) $\left[0, \frac{1}{2}\right]$ c) $[0, 1]$ d) $[1, 2]$
105. The equation $\sqrt{4x+9} - \sqrt{11x+1} = \sqrt{7x+4}$ has
- a) No solution
 b) One solution
 c) Two solutions
 d) More than two solutions

106. $\left|x + \frac{2}{x}\right| < 3$, then x belongs to
a) $(-2, -1) \cup (1, 2)$ b) $(-\infty, -2) \cup (-1, 1) \cup (2, \infty)$
c) $(-2, 2)$ d) $(-3, 3)$
107. If a, b, c are the sides of a triangle, then $\frac{a}{b+c-a} + \frac{b}{c+a-b} + \frac{c}{a+b-c}$ is
a) ≤ 3 b) ≥ 3 c) ≥ 2 d) ≤ 2
108. The minimum value of the sum of the lengths of diagonals of a cyclic quadrilateral of area a^2 square units is
a) $\sqrt{2}a$ b) $2\sqrt{2}a$ c) $2a$ d) None of these
109. $|2x - 3| < |x + 5|$, then x belongs to
a) $(-3, 5)$ b) $(5, 9)$ c) $\left(-\frac{2}{3}, 8\right)$ d) $\left(-8, \frac{2}{3}\right)$
110. The number of real roots of the equation $1 + a_1x + a_2x^2 + \dots + a_nx^n = 0$, where $|x| < \frac{1}{3}$ and $|a_n| < 2$, is
a) n if n is even b) 1 if n is odd c) 0 for any $n \in N$ d) None of these
111. Consider the following statements:
1. If x be real, then $-\frac{9}{2} \leq \frac{3x-4}{x^2+1} \leq \frac{1}{2}$
2. If x is real, then the greatest value of $\frac{x^2+14x+9}{x^2+2x+3}$ is 4
3. If $ax^2 + bx + c = 0$; $a \neq 0, a, b, c \in R$ has no real roots, then $(a + b + c)c > 0$
Which of these is/ are correct?
a) Only (1) b) Only (2) c) Only (3) d) All of these
112. If r is a real number such that $|r| < 1$ and if $a = 5(1 - r)$, then
a) $0 < a < 5$ b) $-5 < a < 5$ c) $0 < a < 10$ d) $0 \leq a < 10$
113. The number of integral roots of the equation $e^{x-8} + 2x - 17 = 0$, is
a) 1 b) 2 c) 4 d) 8
114. The product of real roots of the equation $x^2 + 18x + 30 = 2\sqrt{x^2 + 18x + 45}$, is
a) 720 b) 20 c) 36 d) None of these
115. The set of values of x satisfying $2 \leq |x - 3| < 4$ is
a) $(-1, 1] \cup [5, 7)$ b) $-4 \leq x \leq 2$
c) $-1 < x < 7$ or $x \geq 5$ d) $x < 7$ or $x \geq 5$
116. Let $x = \frac{a+2b}{a+b}$ and $y = \frac{a}{b}$, where a and b are positive integers. If $y^2 > 2$, then
a) $x^2 \leq 2$ b) $x^2 < 2$ c) $x^2 > 2$ d) $x^2 \geq 2$
117. The least value of $5^{\sin x - 1} + 5^{-\sin x - 1}$ is
a) 10 b) $\frac{5}{2}$ c) $\frac{2}{5}$ d) $\frac{1}{5}$
118. If $x^2 + 2x + n > 10$ for all real numbers x , then which of the following conditions is true?
a) $n < 11$ b) $n = 10$ c) $n = 11$ d) $n > 11$
119. The minimum value of $P = bcx + cay + abz$, when $xyz = abc$, is
a) $3abc$ b) $6abc$ c) abc d) $4abc$
120. If $a_i > 0$ for $i = 1, 2, \dots, n$ and $a_1a_2 \dots a_n = 1$, then minimum value of $(1 + a_1)(1 + a_2) \dots (1 + a_n)$ is
a) $2^{n/2}$ b) 2^n c) 2^{2n} d) 1
121. If a, b, c are positive real numbers such that $a + b + c = 2$ then, which one of the following is true?
a) $(2 - a)(2 - b)(2 - c) \geq 8abc$
b) $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} \geq 2$
c) $(2 - a)(2 - b)(2 - c) < 8abc$
d) $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 2$
122. If x, y, z are positive real numbers such that $x^2 + y^2 + z^2 = 27$, then $x^3 + y^3 + z^3$ has
a) Minimum value 81 b) Maximum value 81 c) Minimum value 27 d) Maximum value 27



123. If x satisfies the inequations $2x - 7 < 11, 3x + 4 < -5$, then x lies in the interval
 a) $(-\infty, 3)$ b) $(-\infty, 2)$ c) $(-\infty, -3)$ d) $(-\infty, \infty)$
124. $x^8 - x^5 - \frac{1}{x} + \frac{1}{x^4} > 0$, is satisfied for
 a) Only positive values of x
 b) Only negative values of x
 c) All real numbers except zero
 d) Only for $x > 1$
125. The solution set of the inequation $5^{(1/4)(\log_5 x)^2} \geq 5 x^{(1/5)(\log_5 x)}$, is
 a) $(0, 5^{-2\sqrt{5}}]$ b) $[5^{2\sqrt{5}}, \infty)$ c) $(0, 5^{-2\sqrt{5}}] \cup [5^{2\sqrt{5}}, \infty)$ d) $(0, \infty)$
126. If $a + b = 8$, then ab is greater when
 a) $a = 4, b = 4$ b) $a = 3, b = 5$ c) $a = 6, b = 2$ d) None of these
127. The number of solutions of the equation $\cos x + |x| = 0$ is
 a) 0 b) 1 c) 2 d) 3
128. If $0 < x < \frac{\pi}{2}$, then the minimum value of $\frac{\cos^3 x}{\sin x} + \frac{\sin^3 x}{\cos x}$ is
 a) $\sqrt{3}$ b) $\frac{1}{2}$ c) $\frac{1}{3}$ d) 1
129. If $x^2 + 4ax + 2 > 0$ for all values of x , then a lies in the interval
 a) $(-2, 4)$ b) $(1, 2)$ c) $(-\sqrt{2}, \sqrt{2})$ d) $(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$
130. If a and b are two different positive real numbers then which of the following statement is true?
 a) $2\sqrt{ab} > a + b$ b) $2\sqrt{ab} < a + b$ c) $2\sqrt{ab} = a + b$ d) None of these
131. The number of negative integral solutions of $x^2 \cdot 2^{x+1} + 2^{|x-3|+2} = x^2 \cdot 2^{|x-3|+4} + 2^{x-1}$, is
 a) None b) Only one c) Two d) Four
132. The number of ordered 4-tuples (x, y, z, w) where $x, y, z, w \in [0, 10]$ which satisfy the inequality $2^{\sin^2 x} \times 3^{\cos^2 y} \times 4^{\sin^2 z} \times 5^{\cos^2 w} \geq 120$, is
 a) 81 b) 144 c) 0 d) Infinite
133. If $a > 1, b > 1, c > 1, d > 1$, then the minimum value of $\log_b a + \log_a b + \log_d c + \log_c d$ is
 a) 1 b) 2 c) 3 d) 4
134. The solution set of inequation $\log_{1/3}(2^{x+2} - 4^x) \geq -2$, is
 a) $(-\infty, 2 - \sqrt{13})$ b) $(-\infty, 2 + \sqrt{13})$ c) $(-\infty, 2)$ d) None of these
135. If $\frac{2x}{2x^2+5x+2} > \frac{1}{x+1}$, then
 a) $-2 > x > -1$ b) $-2 \geq x \geq -1$ c) $-2 < x < -1$ d) $-2 < x \leq -1$
136. The number of real solutions of $\log_2 x + |x| = 0$, is
 a) 0 b) 1 c) 3 d) None of these
137. If $xyz = abc$, then the least value of $bcx + cay + abz$ is
 a) $3abc$ b) $6abc$ c) abc d) $4abc$
138. The number of solution(s) of the inequation $\sqrt{3x^2 + 6x + 7} + \sqrt{5x^2 + 10x + 14} \leq 4 - 2x - x^2$, is
 a) 1 b) 2 c) 4 d) Infinitely many
139. A stick of length 20 units is to be divided into n parts so that the product of the lengths of the parts is greater than unity. The maximum possible value of n is
 a) 18 b) 19 c) 20 d) 21
140. If a, b, c are different positive real number such that $(b + c - a), (c + a - b)$ and $(a + b - c)$ are positive, then $(a + b - c)(b + c - a)(c + a - b) - abc$ is
 a) Positive b) Negative c) Non-positive d) Non-negative
141. $\log_{16} x^3 + (\log_2 \sqrt{x})^2 < 1$ iff x lies in
 a) $(2, 16)$ b) $(0, 1/16)$ c) $(1/16, 2)$ d) None of these
142. If $\log_{\cos x} \sin x > 2$ and $0 < x < 3\pi$, then $\sin x$ lies in the interval

- a) $\left[\frac{\sqrt{5}-1}{2}, 1\right]$ b) $\left[0, \frac{\sqrt{5}-1}{2}\right]$ c) $\left[0, \frac{1}{2}\right]$ d) None of these
143. If $f(x) = x^2 + 2bx + 2c^2$ and $g(x) = -x^2 - 2cx + b^2$ such that $\min f(x) > \max g(x)$, then the relation between b and c , is
a) No real value of b and c b) $0 < c < b\sqrt{2}$ c) $|c| < |b|\sqrt{2}$ d) $|c| > |b|\sqrt{2}$
144. If the sum of the greatest integer less than or equal to x and the least integer greater than or equal to x is 5, then the solution set for x is
a) (2, 3) b) (0, 5) c) [5, 6) d) None of these
145. The total number of roots of the equation $|x - x^2 - 1| = |2x - 3 - x^2|$ is
a) 1 b) 2 c) 0 d) Infinitely many
146. For $\frac{|x-1|}{x+2} < 1$, x lies in the interval
a) $(-\infty, -2) \cup \left(-\frac{1}{2}, \infty\right)$ b) $(-\infty, 1) \cup [2, 3]$ c) $(-\infty, -4)$ d) $\left[-\frac{1}{2}, 1\right]$
147. Number of integer solutions of $\frac{x+2}{x^2+1} > \frac{1}{2}$ is
a) 0 b) 1 c) 2 d) 3
148. Solution of the inequality $\tan\left(x + \frac{\pi}{3}\right) \geq 1$ is
a) $\left(n\pi + \frac{\pi}{12}, n\pi + \frac{\pi}{6}\right)$ b) $\left(n\pi - \frac{\pi}{12}, n\pi + \frac{\pi}{6}\right)$ c) $\left(n\pi - \frac{\pi}{6}, n\pi - \frac{\pi}{12}\right)$ d) None of these
149. If $0 < a < 1$, then the solution set of the inequation $\frac{1+(\log_a x)^2}{1+(\log_a x)} > 1$, is
a) (1, 1/a) b) (0, a) c) $(1, 1/a) \cup (0, a)$ d) None of these
150. Let $x = \frac{a+2b}{a+b}$ and $y = \frac{a}{b}$, where a and b are positive integers. If $y^2 > 2$, then
a) $x^2 \leq 2$ b) $x^2 < 2$ c) $x^2 > 2$ d) $x^2 \geq 2$
151. The minimum value of $|\sin x + \cos x + \tan x + \sec x + \operatorname{cosec} x + \cot x|$ is
a) $2\sqrt{2} - 1$ b) $2\sqrt{2} + 1$ c) $\sqrt{2} - 1$ d) $\sqrt{2} + 1$
152. If for $x \in R$, $\frac{1}{3} < \frac{x^2-2x+4}{x^2+2x+4} < 3$, then $\frac{9 \cdot 3^{2x} - 6 \cdot 3^x + 4}{9 \cdot 3^{2x} + 6 \cdot 3^x + 4}$ lies between
a) $\frac{1}{2}$ and 2 b) $\frac{1}{3}$ and 3 c) 0 and 2 d) None of these
153. The minimum value of $4^x + 4^{1-x}$, $x \in R$, is
a) 1 b) 2 c) 4 d) None of these
154. The number of real solutions of the equation $3^{-|x|} - 2^{|x|} = 0$, is
a) 0 b) 1 c) 2 d) None of these
155. The number of real roots of the equation $1 + 3^{x/2} = 2^x$, is
a) 0 b) 1 c) 2 d) None of these
156. If n is even and $n \geq 4$, $x_1, x_2, \dots, x_n \geq 0$ and $x_1 + x_2 + \dots + x_n = 1$, then $P = x_1x_2 + x_2x_3 + \dots + x_n - x_n$ cannot exceed
a) $\frac{1}{n+1}$ b) $\frac{1}{n+2}$ c) $\frac{1}{2n}$ d) None of these
157. The number of real solutions of the equation $e^{-x} = x$, is
a) 0 b) 1 c) 2 d) None of these
158. The solution set contained in R of the inequation $3^x + 3^{1-x} - 4 < 0$, is
a) (1, 3) b) (0, 1) c) (1, 2) d) (0, 2)
159. The solution of the inequation $2x^2 + 3x - 9 \leq 0$ is given by
a) $\frac{3}{2} \leq x \leq 3$ b) $-3 \leq x \leq \frac{3}{2}$ c) $-3 \leq x \leq 3$ d) $\frac{3}{2} \leq x \leq 2$
160. If $0 < \theta < \pi$, then the minimum value of $\sin^5 \theta + \operatorname{cosec}^5 \theta$ is
a) 0 b) 1 c) 2 d) None of these

LINEAR INEQUALITIES

: ANSWER KEY :

1)	a	2)	c	3)	d	4)	b	85)	c	86)	c	87)	d	88)	c
5)	c	6)	a	7)	c	8)	c	89)	c	90)	b	91)	b	92)	d
9)	a	10)	c	11)	b	12)	c	93)	c	94)	a	95)	d	96)	d
13)	b	14)	b	15)	d	16)	c	97)	c	98)	d	99)	d	100)	a
17)	b	18)	c	19)	b	20)	d	101)	d	102)	c	103)	b	104)	a
21)	b	22)	a	23)	a	24)	b	105)	b	106)	a	107)	b	108)	b
25)	b	26)	c	27)	a	28)	b	109)	c	110)	c	111)	d	112)	c
29)	a	30)	b	31)	a	32)	b	113)	a	114)	b	115)	a	116)	b
33)	a	34)	d	35)	c	36)	b	117)	c	118)	d	119)	a	120)	b
37)	a	38)	a	39)	b	40)	b	121)	a	122)	a	123)	c	124)	c
41)	c	42)	c	43)	a	44)	a	125)	c	126)	a	127)	a	128)	d
45)	a	46)	c	47)	b	48)	a	129)	d	130)	b	131)	a	132)	b
49)	a	50)	c	51)	a	52)	d	133)	d	134)	c	135)	c	136)	b
53)	d	54)	d	55)	c	56)	d	137)	a	138)	a	139)	b	140)	b
57)	a	58)	d	59)	c	60)	a	141)	c	142)	b	143)	d	144)	a
61)	b	62)	c	63)	c	64)	c	145)	a	146)	a	147)	d	148)	b
65)	b	66)	d	67)	a	68)	a	149)	c	150)	b	151)	a	152)	b
69)	c	70)	d	71)	a	72)	a	153)	c	154)	b	155)	b	156)	d
73)	c	74)	a	75)	c	76)	c	157)	b	158)	b	159)	b	160)	c
77)	d	78)	d	79)	a	80)	d								
81)	b	82)	a	83)	b	84)	a								



LINEAR INEQUALITIES

: HINTS AND SOLUTIONS :

1 **(a)**
 We have,
 $3^x + 2^{2x} \geq 5^x$
 $\Rightarrow \left(\frac{3}{5}\right)^x + \left(\frac{4}{5}\right)^x \geq 1$
 $\Rightarrow \left(\frac{3}{5}\right)^x + \left(\frac{4}{5}\right)^x \geq \left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2$
 $\Rightarrow x \leq 2 \Rightarrow x \in (-\infty, 2]$ [If $a^x + b^x \geq 1$ and $a^2 + b^2 = 1$, then $x \in (-\infty, 2)$]

2 **(c)**
 $x^2 - 3|x| + 2 < 0$
 $\Rightarrow |x|^2 - 3|x| + 2 < 0$
 $\Rightarrow (|x| - 1)(|x| - 2) < 0$
 $\Rightarrow 1 < |x| < 2$
 $\Rightarrow -2 < x < -1$ or $1 < x < 2$
 $\therefore x \in (-2, -1) \cup (1, 2)$

3 **(d)**
 We have, $2^x + 2^{|x|} \geq 2\sqrt{2}$
 If $x \geq 0$, then $2^x + 2^x \geq 2\sqrt{2}$
 $\Rightarrow 2^x \geq \sqrt{2} \Rightarrow x \geq \frac{1}{2}$
 and if $x < 0$, then $2^x + 2^{-x} \geq 2\sqrt{2}$
 $\Rightarrow t + \frac{1}{t} \geq 2\sqrt{2}$ (where $t = 2^x$)
 $\Rightarrow t^2 - 2\sqrt{2}t + 1 \geq 0$
 $\Rightarrow (t - (\sqrt{2} - 1))(t - (\sqrt{2} + 1)) \geq 0$
 $\Rightarrow t \leq \sqrt{2} - 1$ or $t \geq \sqrt{2} + 1$
 But $t > 0$

$$\Rightarrow 0 < 2^x \leq \sqrt{2} - 1$$

$$\text{Or } 2^x \geq \sqrt{2} + 1$$

$$\Rightarrow -\infty < x \leq \log_2(\sqrt{2} - 1)$$

$$\text{Or } x \geq \log_2(\sqrt{2} + 1)$$

Which is not possible, because $x > 0$

$$\therefore x \in (-\infty, \log_2(\sqrt{2} - 1)) \cup \left[\frac{1}{2}, \infty\right)$$

4 **(b)**
 Using G.M. \leq A.M., we have
 $\sin x_i \cos x_{i+1} \leq \frac{\sin^2 x_i + \cos^2 x_{i+1}}{2}$ for $i = 1, 2, 3, \dots, n$,
 where $x_{n+1} = x_1$
 $\therefore \sin x_1 \cos x_2 + \sin x_2 \cos x_3 + \dots + \sin x_n \cos x_{n+1}$
 $\leq \frac{\sin^2 x_1 + \cos^2 x_2}{2} + \frac{\sin^2 x_2 + \cos^2 x_3}{2} + \dots + \frac{\sin^2 x_n + \cos^2 x_1}{2}$
 $\Rightarrow \sin x_1 \cos x_2 + \sin x_2 \cos x_3 + \dots + \sin x_n \cos x_1 \leq \frac{n}{2}$

5 **(c)**
 We have, $x^2 + (a + b)x + ab < 0$
 $\Rightarrow (x + a)(x + b) < 0$
 $\Rightarrow -b < x < -a$

6 **(a)**
 Given, $\log_{10}(x^3 + y^3) - \log_{10}(x^2 + y^2 - xy) \leq 2$
 $\Rightarrow \log_{10} \frac{(x^3 + y^3)}{x^2 + y^2 - xy} \leq 2$
 $\Rightarrow \log_{10}(x + y) \leq 2 \Rightarrow x + y \leq 100$
 Using AM \geq GM

$$\begin{aligned} \therefore \frac{x+y}{2} &\geq \sqrt{xy} \\ \Rightarrow \sqrt{xy} &\leq \frac{x+y}{2} \leq \frac{100}{2} \\ \Rightarrow xy &\leq 2500 \end{aligned}$$

7 (c)

By trial,

$$3^{x/2} + 2^x \leq 25 \text{ for } x = 1, 2, 3, 4$$

$$\text{But } 3^{x/2} + 2^x > 25 \text{ for } x > 4$$

Hence, solution set for $3^{x/2} + 2^x > 25$ is $(4, \infty)$

8 (c)

Since,

$$\text{AM} \geq \text{GM}$$

$$\Rightarrow \frac{a+b}{2} \geq \sqrt{ab}$$

$$\Rightarrow \frac{a+b}{2} \geq \sqrt{4} \quad (\because ab = 4, \text{ given})$$

$$\Rightarrow a + b \geq 4$$

9 (a)

When $x > 0$, $P_n(x) > 0$ and so $P_n(x) = 0$ can have no positive real roots

Now,

$$P_n(x) = 1 + 2x + 3x^2 + \dots + (n+1)x^n$$

$$\Rightarrow xP_n(x) = x + 2x^2 + 3x^3 + \dots + nx^n + (n+1)x^{n+1}$$

$$\begin{aligned} \Rightarrow (1-x)P_n(x) &= 1 + x + x^2 + \dots + x^n \\ &\quad - (n+1)x^{n+1} \end{aligned}$$

$$\Rightarrow P_n(x) = \frac{1 - (n+2)x^{n+1} + (n+1)x^{n+2}}{(1-x)^2}$$

For negative values of x , $P_n(x)$ will vanish whenever

$$f(x) = 1 - (n+2)x^{n+1} + (n+1)x^{n+2} = 0$$

Now,

$$f(-x) = 1 - (n+2)(-1)^{n+1}x^{n+1} + (n+1)(-1)^{n+2}x^{n+2} \dots \text{(i)}$$

If n is even, there is no change of sign in this expression and so there is no negative real root of $f(x)$

10 (c)

$$(x-1)(x^2 - 5x + 7) < (x-1)$$

$$\Rightarrow (x-1)(x^2 - 5x + 6) < 0$$

$$\Rightarrow (x-1)(x-2)(x-3) < 0$$

$$\therefore x \in (-\infty, 1) \cup (2, 3)$$

11 (b)

Using A.M. \geq G.M., we have

$$\frac{x}{n} \geq \{\log_2 2 \times \log_2 2^2 \times \dots \times \log_2 2^{n+1} 2^n\}^{1/n}$$

$$\Rightarrow x \geq n(\log_2 2)^{1/n}$$

$$\Rightarrow x \geq n \left(\frac{1}{n+1} \log_2 2 \right)^{1/n} \Rightarrow x \geq n \left(\frac{1}{n+1} \right)^{1/n}$$

12 (c)

We have,

$$\frac{1}{2} \left| a + \frac{11}{a} \right| \geq \sqrt{11}, \text{ equality holding iff } a = \pm\sqrt{11}$$

$$\therefore |x| \geq \sqrt{11}, |y| \geq \sqrt{11}, |z| \geq \sqrt{11}, |t| \geq \sqrt{11}$$

$$\text{Let } x \geq 0, \text{ then } x \geq \sqrt{11}, y \geq \sqrt{11}, z \geq$$

$$\sqrt{11} \text{ and } t \geq \sqrt{11}$$

$$\text{Now, } y - \sqrt{11} = \frac{1}{2} \left(\frac{11}{x} + x \right) - \sqrt{11}$$

$$\Rightarrow y - \sqrt{11} = \frac{(x - \sqrt{11})^2}{2x} = \left(\frac{x - \sqrt{11}}{2x} \right) (x - \sqrt{11})$$

$$\Rightarrow y - \sqrt{11} = \frac{1}{2} \left(1 - \frac{\sqrt{11}}{x} \right) (x - \sqrt{11})$$

$$< (x - \sqrt{11})$$

$$\Rightarrow y < x \text{ i.e. } x > y$$

Similarly, we have

$$y > z, z > t \text{ and } t > x \Rightarrow y > x$$

$$\therefore x = y = z = t = \sqrt{11} \text{ is the only solution for } x > 0$$

We observe that (x, y, z, t) is a solution iff

$(-x, -y, -z, -t)$ is a solution

Thus, $x = y = z = t = -\sqrt{11}$ is the only other solution

13 (b)

$$\text{Given, } 3 + \frac{3}{3^x} - 4 < 0 \Rightarrow 3^{2x} + 3 - 4.3^x < 0$$

$$\Rightarrow (3^x - 1)(3^x - 3) < 0$$

$$1 < 3^x < 3 \Rightarrow 0 < x < 1$$

$$\therefore \text{The solution set is } (0, 1)$$

14 (b)

We have, $|a| \leq 1$ and $a + b = 1$

i.e. $-1 \leq a \leq 1$ and $b = 1 - a$

$$\Rightarrow -1 \leq a \leq 1 \text{ and } 0 \leq b \leq 2 \Rightarrow -2 \leq ab \leq 2$$

...(i)

Now,

$$ab \leq \left(\frac{a+b}{2} \right)^2 \Rightarrow ab \leq \frac{1}{4} \quad \dots \text{(ii)}$$

From (i) and (ii), we have

$$-2 \leq ab \leq \frac{1}{4} \Rightarrow ab \in \left[-\frac{2}{4}, \frac{1}{4} \right]$$

15 (d)

Given, $\sqrt{(3x+1)^2} < (2-x)$
 $\Rightarrow (3x+1) < 2-x$
 $\Rightarrow 3x+1 < 2-x \Rightarrow x < \frac{1}{4}$

16 (c)

Given, inequality can be rewritten as $\left(\frac{5}{13}\right)^x + \left(\frac{12}{13}\right)^x \geq 1$

$\therefore \cos^x \alpha + \sin^x \alpha \geq 1$

Where, $\cos \alpha = \frac{5}{13}$

If $x = 2$, the above equality holds

If $x < 2$ both $\cos \alpha$ and $\sin \alpha$ increases in positive fraction

Hence, above inequality holds for $x \in (-\infty, 2]$

17 (b)

Let $f(x) = \log_e \frac{x-2}{x-3}$

$f(x)$ is defined either $(x-2) > 0, (x-3) > 0$ or $(x-2) < 0$

$(x-3) < 0$ or $x \neq 2, 3$

$\Rightarrow f(x)$ is defined either $x > 3$ or $x < 2$ or $x \neq 2, 3$

ie, $x \in (-\infty, 2) \cup (3, \infty)$

18 (c)

$3 \leq 3t - 18 \leq 18$

$\Rightarrow 21 \leq 3t \leq 36$

$\Rightarrow 7 \leq t \leq 12$

$\Rightarrow 8 \leq t + 1 \leq 13$

19 (b)

$\because f(-1) < 1 \Rightarrow a - b + c < 1 \dots(i)$

and $f(1) > -1, f(3) < -4$, then

$a + b + c > -1 \dots(ii)$

$9a + 3b + c < -4 \dots(iii)$

From Eq. (ii),

$-a - b - c < 1 \dots(iv)$

On solving Eqs. (i), (iii) and (iv), we get $a < -\frac{1}{8} \Rightarrow$

a is negative

20 (d)

Given, $\frac{2x+3}{2x-9} < 0$

$\Rightarrow 2x+3 < 0$ and $2x-9 > 0$

Or $2x+3 > 0$ and $2x-9 < 0$ and $x \neq \frac{9}{2}$

$\Rightarrow x < -\frac{3}{2}$ and $x > \frac{9}{2}$ or $x > -\frac{3}{2}$ and $x < \frac{9}{2}$ and $x \neq \frac{9}{2}$

$\Rightarrow x \in \left(-\frac{3}{2}, \frac{9}{2}\right)$

21 (b)

We have,

$\sqrt{x^2 + \sqrt{x^2 + 11}} - \sqrt{x^2 - \sqrt{x^2 + 11}} = 4$

Putting $x^2 + 11 = t^2$, we get

$\sqrt{t^2 + t - 11} + \sqrt{t^2 - t - 11} = 4 \dots(i)$

But, $(t^2 + t - 11) - (t^2 - t - 11) = 2t \dots(ii)$

Dividing (ii) by (i), we get

$\sqrt{t^2 + t - 11} - \sqrt{t^2 - t - 11} = \frac{t}{2} \dots(iii)$

Adding (i) and (iii), we get

$2\sqrt{t^2 + t - 11} = 4 + \frac{t}{2}$

$\Rightarrow t^2 + t - 11 = 4 + t + \frac{t^2}{16}$

$\Rightarrow t^2 = 16 \Rightarrow t = 4 \quad [\because t = \sqrt{x^2 + 11} > 0]$

$\therefore x = \pm\sqrt{5}$

22 (a)

We have,

$\log_{x-3}(x^3 - 3x^2 - 4x + 8) = 3 \dots(i)$

$\Rightarrow x^3 - 3x^2 - 4x + 8 = (x-3)^3$

$\Rightarrow 6x^2 - 31x + 35 = 0 \Rightarrow (3x-5)(2x-7) = 0$

$\Rightarrow x = \frac{5}{3}, \frac{7}{2}$

The equation (i) exists, if

$x-3 > 0, x-3 \neq 1$ and $x^3 - 3x^2 - 4x + 8 > 0$

Clearly, $x = \frac{7}{2}$ satisfies these conditions

23 (a)

Curves $y = \log_{0.5} x$ and $y = |x|$ intersect at one point in first quadrant. So, the equation $\log_{0.5} x = |x|$ has one real root

25 (b)

$\cos^x \alpha + \sin^x \alpha \geq 1$

Equality holds when $x = 2$

If $x < 2$, both $\cos \alpha$ and $\sin \alpha$ are increasing

$\therefore \cos^x \alpha + \sin^x \alpha > 1$, if $x < 2$

If $x > 2$, then $\cos^x \alpha + \sin^x \alpha < 1$

$\therefore x \in (-\infty, 2]$

27 (a)

We have,

$2 \cos(e^x) = 3^x + 3^{-x}$

We observe that $2 \cos(e^x) < 2$ and $3^x + 3^{-x} \geq 2$.
So, the given equation has no solution

28 **(b)**

Graphs of $y = 1 - x$ and $y = [\cos x]$ cut each other at point $(0, 1)$ and at a point whose x -coordinate lie in $(\pi/2, \pi)$. So, the given equation has two real roots

30 **(b)**

If $a^2 + b^2 = 1$, then $a^x + b^x \geq 1$ is true for all $x \in (-\infty, 2]$

$$\therefore (\sin \alpha)^x + (\cos \alpha)^x \geq 1 \Rightarrow x \in (-\infty, 2]$$

31 **(a)**

If $f(x) > 0$, then $D < 0$

$$4a^2 - 4(10 - 3a) < 0$$

$$\Rightarrow (a + 5)(a - 2) < 0$$

$$\Rightarrow -5 < a < 2$$

32 **(b)**

The given inequality is

$$49.4 - \left(\frac{27-x}{10}\right) < 47.4 - \left(\frac{27-9x}{10}\right)$$

$$\Rightarrow 49.4 - 47.4 < \left(\frac{27-x}{10}\right) - \left(\frac{27-9x}{10}\right)$$

$$\Rightarrow 2 < \frac{8x}{10} \Rightarrow x > \frac{5}{2}$$

\therefore Least integer is 3

33 **(a)**

Since, AM \geq GM

$$\therefore \frac{a^2 + b^2}{2} \geq \sqrt{a^2 b^2} = ab, \frac{b^2 + c^2}{2} \geq bc$$

$$\text{and } \frac{c^2 + a^2}{2} \geq ca$$

On adding, we get

$$a^2 + b^2 + c^2 \geq ab + bc + ca$$

\Rightarrow (a) holds

$$\text{Next, } \frac{b+c}{2} \geq \sqrt{bc}, \frac{c+a}{2} \geq \sqrt{ca}, \frac{a+b}{2} \geq \sqrt{ab}$$

$$\Rightarrow \left(\frac{b+c}{2}\right)\left(\frac{c+a}{2}\right)\left(\frac{a+b}{2}\right) \geq \sqrt{a^2 b^2 c^2}$$

$$\Rightarrow (b+c)(c+a)(a+b) \geq 8abc$$

\Rightarrow (b) does not hold

$$\text{Again, } \frac{1}{3}\left(\frac{a}{b} + \frac{b}{c} + \frac{c}{a}\right) \geq \left(\frac{a}{b} \cdot \frac{b}{c} \cdot \frac{c}{a}\right)^{1/3}$$

$$\Rightarrow \frac{a}{b} + \frac{b}{c} + \frac{c}{a} \geq 3$$

\Rightarrow (c) does not hold

$$\text{Again, } \frac{a^3 + b^3 + c^3}{3} \geq (a^3 b^3 c^3)^{1/3}$$

$$\Rightarrow a^3 + b^3 + c^3 \geq 3abc$$

\Rightarrow (d) does not hold

34 **(d)**

If s is the semi-perimeter of a cyclic quadrilateral of sides a, b, c and d units in length, then its area A is given by

$$A = \sqrt{(s-a)(s-b)(s-c)(s-d)}$$

Using A. M. \geq G. M., we have

$$\frac{s-a + s-b + s-c + s-d}{4} \geq \{(s-a)(s-b)(s-c)(s-d)\}^{1/4}$$

$$\Rightarrow \frac{4s - 2s}{4} \geq \sqrt{A} \Rightarrow 2s \geq 4\sqrt{A}$$

Hence, the least perimeter is $4\sqrt{A}$

35 **(c)**

Two curves

$$y = [\sin x + \cos x]$$

$$\text{and, } y = 3 + [-\sin x] + [-\cos x]$$

$$= 1 + [\sin x] + [\cos x]$$

intersect at infinitely many points in $[0, 2\pi]$

So, the given equation has infinitely many solutions

36 **(b)**

Two curves $y = 3^{|x|}$ and $y = |2 - |x||$ intersect at two points only. So, the equation $3^{|x|} = |2 - |x||$ has only two real roots

37 **(a)**

Since, angle C is obtuse, angle A and B are acute

$$\therefore \tan C < 0 \text{ and } \tan A > 0, \tan B > 0$$

$$\text{Now, } \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\Rightarrow \tan(\pi - C) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\Rightarrow -\tan C = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$\Rightarrow 1 - \tan A \tan B > 0$ (\because Numerator are positive)

$\Rightarrow \tan A \tan B < 1$

38 (a)

We have,

$$x + y + z = 4 \text{ and } x^2 + y^2 + z^2 = 6$$

$$\Rightarrow y + z = 4 - x \text{ and } y^2 + z^2 = 6 - x^2$$

$$\begin{aligned} \therefore yz &= \frac{1}{2}\{(y+z)^2 - (y^2 + z^2)\} \\ &= \frac{1}{2}\{(4-x)^2 - (6-x^2)\} \end{aligned}$$

$$\Rightarrow yz = x^2 - 4x + 5$$

Thus, y and z are roots of the equation

$$t^2 - (4-x)t + x^2 - 4x + 5 = 0$$

As y, z are real

$$\therefore (4-x)^2 - 4(x^2 - 4x + 5) \geq 0$$

$$\Rightarrow 3x^2 - 8x + 4 \leq 0 \Rightarrow \frac{2}{3} \leq x \leq 2$$

40 (b)

We have,

$$3^{x/2} + 2^x > 25 \Rightarrow 3^{x/2} + 4^{x/2} > 25$$

Clearly, $x \in (4, \infty)$ satisfies the above inequation

41 (c)

We have,

$$27^{1/x} + 12^{1/x} = 2 \times 8^{1/x}$$

$$\Rightarrow 3^{3/x} + 2^{2/x} \times 3^{1/x} = 2 \times 2^{3/x}$$

$$\Rightarrow \left(\frac{3}{2}\right)^{3/x} + \left(\frac{3}{2}\right)^{1/x} = 2$$

$$\Rightarrow y^3 + y - 2 = 0, \text{ where } y = \left(\frac{3}{2}\right)^{1/x}$$

$$\Rightarrow (y-1)(y^2 + y - 2) = 0$$

$$\Rightarrow y = 1, y = -2 \Rightarrow \left(\frac{3}{2}\right)^{1/x} = 1 \Rightarrow \left(\frac{3}{2}\right)^{1/x} = \left(\frac{3}{2}\right)^0$$

But, there is no value of x for which $\frac{1}{x}$ is zero

Hence, the given equation has no solution

42 (c)

Let x_1, x_2, x_3 and x_4 be four positive roots of the equation $x^4 - 8x^3 + bx^2 + cx + 16 = 0$. Then,

$$x_1 + x_2 + x_3 + x_4 = 8 \text{ and } x_1 x_2 x_3 x_4 = 16$$

$$\Rightarrow \frac{x_1 + x_2 + x_3 + x_4}{4} = 2 \text{ and } (x_1 x_2 x_3 x_4)^{1/4} = 2$$

\Rightarrow A.M. and G.M. of x_1, x_2, x_3 and x_4 are equal

$$\Rightarrow x_1 = x_2 = x_3 = x_4$$

$$\Rightarrow x_1 = x_2 = x_3 = x_4 = 2$$

$$\therefore x^4 - 8x^3 + bx^2 + cx + 16 = (x-2)^4$$

$$\Rightarrow b = {}^4C_2 \times 2^2 = 24 \text{ and } c = -{}^4C_3 \times 2^3 = -32$$

43 (a)

We have, $3 < |x| < 6 \Rightarrow -6 < x < -3$ or $3 < x < 6$

$$\therefore x \in (-6, -3) \cup (3, 6)$$

44 (a)

We have,

$$\begin{aligned} a^4 + b^4 - a^3 b - ab^3 &= a^3(a-b) - b^3(a-b) \\ &= (a^3 - b^3)(a-b) \end{aligned}$$

$$\begin{aligned} \Rightarrow a^4 + b^4 - a^3 b - ab^3 &> 0 \quad \left[\begin{array}{l} \because a^3 - b^3 \text{ and} \\ a - b \text{ are of the same sign} \end{array} \right] \end{aligned}$$

$$\Rightarrow a^4 + b^4 > a^3 b + ab^3$$

45 (a)

The given inequation is

$$4^{-x+0.5} - 7 \cdot 2^{-x} < 4, x \in R$$

$$\text{Let } 2^{-x} = t$$

$$\therefore 2t^2 - 7t < 4$$

$$\Rightarrow 2t^2 - 7t - 4 < 0$$

$$\Rightarrow (2t+1)(t-4) < 0$$

$$\Rightarrow -\frac{1}{2} < t < 4$$

$$\Rightarrow 0 < t < 4 \quad (\because t = 2^{-x} > 0)$$

$$\Rightarrow 0 < 2^{-x} < 2^2$$

As 2^x is an increasing function $-x < 2$ or $x > -2$

$$\therefore x = (-2, \infty)$$

46 (c)

Given condition are $\frac{a}{b} > 1$ and $\frac{a}{c} < 0$

1. $a > 0$ iff $c < 0$ and also $b > 0$

2. $a < 0$ iff $c > 0$ and also $b < 0$

47 (b)

Proceeding as in the solution of Q. no. 10, we have

$$(a+b)(b+c)(c+a) \geq 8abc$$

$$\Rightarrow (p-a)(p-b)(p-c) \geq 8abc \quad [\because a+b+c = p]$$

48 (a)

We have,

$$\frac{(1+e^{x^2})\sqrt{1+x^2}}{\sqrt{1+x^4}-x^2} = 1 + \cos x$$

$$\Rightarrow (1+e^{x^2})\sqrt{1+x^2}(\sqrt{1+x^4}+x^2) = 1 + \cos x$$

Clearly, $LHS \geq 2$ and $RHS \leq 2$. So, the equation exists when each side is equal to 2. This is for $x = 0$ only. Hence, it has only one solution

50 (c)

Let $f(x) = c_0 + c_1x + c_2x^2 + \dots + c_nx^n$. Then,
 $f(b) - f(a) = 1$
 $\Rightarrow c_1(b-a) + c_2(b^2 - a^2) + \dots + c_n(b^n - a^n)$
 $= 1$
 $\Rightarrow (b-a)\{c_1 + c_2(b+a) + \dots$
 $+ c_n(b^{n-1} + b^{n-2}a + \dots + a^{n-1})\}$
 $= 1$
 $\Rightarrow (b-a)l = 1$, where l is an integer
 $\Rightarrow b-a = \pm 1$

52 (d)

Using, $AM > GM$

$$\therefore \frac{a+b+c}{3} > \sqrt[3]{abc}$$

$$\Rightarrow a+b+c > 3 \quad \dots(i)$$

[$\because abc = 1$ given]

Also, $GM > HM$

$$\sqrt[3]{abc} > \frac{3}{\frac{1}{a} + \frac{1}{b} + \frac{1}{c}}$$

$$\Rightarrow (1)^{1/3} > \frac{3abc}{bc+ac+ab}$$

$$\Rightarrow ab+bc+ac > 3 \quad \dots(ii)$$

\therefore From Eqs. (i) and (ii), we get

$$a+b+c+ab+bc+ac > 6$$

53 (d)

We have,

$$\sin(2^x) \cos(2^x) = \frac{2^x + 2^{-x}}{2}$$

$$\Rightarrow \sin(2^{x+1}) = 2^x + 2^{-x}$$

Clearly, $RHS \geq 2$ and LHS lies between -1 and 1 .

So, the given equation has no solution

54 (d)

$$x^{12} - x^9 + x^4 - x + 1 > 0$$

When $0 < x < 1$; $x^4 > x^9$ and $1 > x$

$$\therefore x^{12} + (x^4 - x^9) + (1 - x) > 0$$

\Rightarrow Positive for all x

Again, when $x > 1$:

$$x^{12} - x^9 + x^4 - x + 1 > 0$$

\therefore Largest interval $(0, \infty)$, also the above inequality is true for $x < 0$

56 (d)

$\therefore AM \geq GM$

$$\Rightarrow \frac{\frac{\cos^3 x}{\sin x} + \frac{\sin^3 x}{\cos x}}{2} \geq \left(\frac{\cos^3 x}{\sin x} \cdot \frac{\sin^3 x}{\cos x} \right)^{1/2}$$

$$\Rightarrow \frac{\cos^3 x}{\sin x} + \frac{\sin^3 x}{\cos x} \geq 2 \sin x \cos x \geq 1$$

Hence, option (d) is correct

57 (a)

We have,

$$\sqrt{3x^2 + 6x + 7} = \sqrt{3(x+1)^2 + 4} \geq 2($$

$$= 2 \text{ when } x = -1)$$

$$\sqrt{5x^2 + 10x + 14} = \sqrt{5(x+1)^2 + 9} \geq 3($$

$$= 3 \text{ when } x = -1)$$

and,

$$4 - 2x - x^2 = 5 - (x+1)^2 \leq 5 (= 5 \text{ when } x$$

$$= -1)$$

Thus, $LHS \geq 5$ and $RHS \leq 5$

So, the given equation is valid when each sides is equal to 5.

This happens only when $x = -1$

Hence, the given equation has only one solution

58 (d)

$$||x| - 1| < |1 - x|$$

Case I $x \geq 0$

\therefore Inequality (i) becomes $|x - 1| < x - 1$ or $|1 - x| < 1 - x$ which is not satisfied by any x , because

$$|a| \geq \forall a \in R$$

Case II $-1 \leq x < 0$

\therefore Inequality (i) becomes $|-1 - x| < 1 - x$ or $|x + 1| < 1 - x$

Or $x + 1 < 1 - x$ or $x < 0$

Thus, inequality (i) is satisfied for $-1 \leq x < 0$

Case III $x < -1$

Inequality (i) becomes $|-1 - x| < 1 - x \Rightarrow$
 $|1 + x| < 1 - x$

$$\Rightarrow -(1 + x) < 1 - x \Rightarrow -2 < 0, \text{ which is true}$$

So, solution set is $(-\infty, 0)$

59 (c)

Minimum value of $f(x)$

Is attained at $x = 3$

\therefore Minimum value of $f(x) = 7$

- 60 (a)
 $\frac{x+11}{x-3} > 0$
 $\Rightarrow (x-3)(x+11) > 0$
 $\Rightarrow x < -11, x > 3$
 $\Rightarrow x \in (-\infty, -11) \cup (3, \infty)$
- 61 (b)
 $2x - 1 = |x + 7| = \begin{cases} x + 7, & \text{if } x \geq -7 \\ -(x + 7), & \text{if } x < -7 \end{cases}$
 \therefore If $x \geq -7, 2x - 1 = x + 7 \Rightarrow x = 8$
 If $x < -7, 2x - 1 = -(x + 7)$
 $\Rightarrow 3x = -6$
 $\Rightarrow x = -2$, which is not possible
- 62 (c)
 Let $f(x) = x^4 - 4x - 1$. Then, the number of changes of signs in $f(x)$ is 1. Therefore, $f(x)$ can have at most one positive real root
 We have,
 $f(2) > 0$ and $f(0) < 1$
 Therefore, $f(x)$ has one positive real root between 1 and 2
- 63 (c)
 $\log_{\sin(\frac{\pi}{3})}(x^2 - 3x + 2) \geq 2$
 $\Rightarrow x^2 - 3x + 2 \leq \frac{3}{4}$ (If $\log_a b = c \Rightarrow b = c^a$)
 $\Rightarrow x^2 - 3x + \frac{5}{4} \leq 0$
 $\Rightarrow 4x^2 - 12x + 5 \leq 0$
 $\Rightarrow (2x - 5)(2x - 1) \leq 0$
 $\Rightarrow \frac{1}{2} \leq x \leq \frac{5}{2}$... (i)
 Also, $x^2 - 3x + 2 > 0$
 $\Rightarrow (x - 1)(x - 2) > 0$
 $\Rightarrow x < 1$ or $x > 2$... (ii)
 From relation (i) and (ii), we get
 $x \in \left[\frac{1}{2}, 1\right) \cup \left(2, \frac{5}{2}\right]$
- 64 (c)
 Since, $(a - b)^2 + (b - c)^2 + (c - a)^2 \geq 0$
 $\Rightarrow 2(a^2 + b^2 + c^2) \geq 2(ab + bc + ca)$

$$\Rightarrow \frac{a^2 + b^2 + c^2}{(ab + bc + ca)} \geq 1$$

$$\Rightarrow \frac{a^2 + b^2 + c^2}{ab + bc + ca} + 2 \geq 3$$

Hence, option (c) is correct

- 65 (b)
 Given, $(x - 1)(x^2 - 5x + 7) < (x - 1)$
 $\Rightarrow (x - 1)(x^2 - 5x + 6) < 0$
 $\Rightarrow (x - 1)(x - 2)(x - 3) < 0$
 $\Rightarrow x \in (-\infty, 1) \cup (2, 3)$
- 66 (d)
 Given inequalities are
 $x^2 - 3x - 10 < 0$ and $10x - x^2 - 16 > 0$
 $\Rightarrow (x + 2)(x - 5) < 0$ and $(x - 2)(x - 8) < 0$
 $\Rightarrow x \in (-2, 5)$ and $x \in (2, 8)$
 $\Rightarrow x \in (2, 5)$
- 67 (a)
 $\log_{1/3}(x^2 + x + 1) < -1 = \log_{1/3}\left(\frac{1}{3}\right)^{-1}$
 $\Rightarrow x^2 + x + 1 > \left(\frac{1}{3}\right)^{-1}$
 (\because where $0 < a < 1$, then $\log_a x < \log_a y \Rightarrow x > y$)
 $\Rightarrow x^2 + x - 2 > 0 \Rightarrow (x + 2)(x - 1) > 0$
 $\Rightarrow x \in (-\infty, -2) \cup (1, \infty)$
- 68 (a)
 Let (α, β) be a solution of the system for some a . Then, $(-\alpha, \beta)$ is also a solution. So, the system will have unique solution only if
 $\alpha = -\alpha \Rightarrow \alpha = 0$
 Putting $x = \alpha = 0$ and $y = \beta$ in $x^2 + y^2 = 1$, we get $\beta = \pm 1$
 Putting $x = \alpha = 0$ and $y = \beta$ in $2^{|x|} + |x| = y + x^2 + a$, we get
 $\beta + a = 1 \Rightarrow a = 1 - \beta$
 $\therefore a = 0$ when $\beta = 1$ and $a = 2$ when $\beta = -1$
CASE I When $a = 0$
 In this case, given equations become
 $2^{|x|} + |x| = y + x^2$ and $x^2 + y^2 = 1$
 Now, $x^2 + y^2 = 1 \Rightarrow |x| \leq 1$ and $|y| \leq 1$
 $\therefore 2^{|x|} + |x| = y + x^2$ and $1 + x^2 \geq y + x^2$
 $\Rightarrow 2^{|x|} + |x| \leq 1 + x^2$
 $\Rightarrow 2^{|x|} + |x| \leq 1 + |x|$ [$\because x^2 \leq |x|$ when $|x| < 1$]
 $\Rightarrow x = 0$
 Putting $x = 0$ in $2^{|x|} + |x| = y + x^2$, we get $y = 1$



Thus, for $a = 0$, the system has unique solution $(0, 1)$

CASE II When $a = 2$

In this case, the system of equation is $2^{|x|} + |x| = y + x^2 + 2$ and $x^2 + y^2 = 1$

Clearly, $(0, -1)$, $(1, 0)$ and $(-1, 0)$ satisfy these equations.

So, the system does not have unique solution

69 (c)

We have,

$$\log_{1/3}(x^2 + x + 1) + 1 > 0$$

$$\Rightarrow \log_{1/3}(x^2 + x + 1) > -1$$

$$\Rightarrow x^2 + x + 1 < \left(\frac{1}{3}\right)^{-1}$$

$$\Rightarrow x^2 + x + 1 < 3$$

$$\Rightarrow x^2 + x - 2 < 0 \Rightarrow (x + 2)(x - 1) < 0 \Rightarrow x \in (-2, 1)$$

70 (d)

We have,

$$x^{(\log_{10} x)^2 - 3(\log_{10} x) + 1} > 10^3$$

$$\Rightarrow (\log_{10} x)^2 - 3(\log_{10} x) + 1 > \log_x 10^3$$

$$\Rightarrow (\log_{10} x)^2 - 3(\log_{10} x) + 1 > \frac{3}{\log_{10} x}$$

$$\Rightarrow \frac{(\log_{10} x)^3 - 3(\log_{10} x)^2 + (\log_{10} x) - 3}{\log_{10} x} > 0$$

$$\Rightarrow \frac{\{(\log_{10} x)^2 + 1\}(\log_{10} x - 3)}{\log_{10} x} > 0$$

$$\Rightarrow \frac{(\log_{10} x - 3)}{(\log_{10} x - 0)} > 0$$

$$\Rightarrow \log_{10} x < 0 \text{ or } \log_{10} x > 3$$

$$\Rightarrow x < 1 \text{ or } x > 10^3$$

$$\Rightarrow x \in (0, 1) \cup (10^3, \infty) [\because \log_{10} x \text{ is defined for } x > 0]$$

71 (a)

$$\text{Given, } \frac{3-|x|}{4-|x|} \geq 0$$

$$\Rightarrow 3 - |x| \leq 0 \text{ and } 4 - |x| < 0$$

$$\text{Or } 3 - |x| \geq 0 \text{ and } 4 - |x| > 0$$

$$\Rightarrow |x| \geq 3 \text{ and } |x| > 4$$

$$\text{Or } |x| \leq 3 \text{ and } |x| < 4$$

$$\Rightarrow |x| > 4 \text{ or } |x| \leq 3$$

$$\Rightarrow (-\infty, -4) \cup [-3, 3] \cup (4, \infty)$$

72 (a)

$$\text{Now, } 3(a^2 + b^2 + c^2) - (a + b + c)^2$$

$$= 2(a^2 + b^2 + c^2 - bc - ca - ab)$$

$$= (b - c)^2 + (c - a)^2 + (a - b)^2 \geq 0$$

$$\Rightarrow 3(a^2 + b^2 + c^2) \geq (a + b + c)^2 > 9$$

$$\Rightarrow a^2 + b^2 + c^2 > 3 \Rightarrow (a) \text{ holds}$$

$$\text{Now, } a^6 + b^6 \geq 12a^2b^2 - 64$$

$$\text{If } a^6 + b^6 + 64 \geq 12a^2b^2$$

$$\text{ie, } a^6 + b^6 + 2^6 \geq 3 \cdot 2^2 \cdot a^2b^2$$

$$\text{ie, if } \frac{a^6 + b^6 + 2^6}{3} \geq (2^6 a^6 b^6)^{1/3} \quad (\because \text{AM} \geq \text{GM})$$

$$\Rightarrow (b) \text{ does not hold}$$

Again, since $\text{AM} \geq \text{HM}$

$$\therefore \frac{a + b + c}{3} \geq \frac{3}{\frac{1}{a} + \frac{1}{b} + \frac{1}{c}}$$

$$\Rightarrow \frac{a}{3} \geq \frac{3}{\frac{1}{a} + \frac{1}{b} + \frac{1}{c}}$$

$$\Rightarrow \frac{1}{a} + \frac{1}{b} + \frac{1}{c} \geq \frac{9}{a}$$

$$\Rightarrow (c) \text{ does not hold}$$

73 (c)

Using $\text{A.M.} \geq \text{G.M.}$, we have

$$\frac{a_1 + a_2 + a_3}{3} \geq (a_1 a_2 a_3)^{1/3} \text{ and } \frac{1}{3} \left(\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3} \right) \geq \left(\frac{1}{a_1 a_2 a_3} \right)^{1/3}$$

$$\Rightarrow (a_1 + a_2 + a_3) \left(\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3} \right) \geq 9$$

74 (a)

We have,

$$x - \frac{1}{x} = B \text{ and } x^2 + \frac{1}{x^2} = A$$

$$\therefore \left(x - \frac{1}{x} \right)^2 = B^2$$

$$\Rightarrow A - 2 = B^2 \Rightarrow A = B^2 + 2 \Rightarrow \frac{A}{B} = B + \frac{2}{B}$$

But, $\text{A.M.} \geq \text{G.M.}$

$$\Rightarrow B + \frac{2}{B} \geq 2 \sqrt{B \times \frac{2}{B}} \Rightarrow B + \frac{2}{B} \geq 2\sqrt{2} \Rightarrow \frac{A}{B} \geq 2\sqrt{2}$$

Hence, the minimum value of $\frac{A}{B}$ is $2\sqrt{2}$

75 (c)

As discussed in the above problem, if n is odd, there is one change of sign in (i). Therefore, $f(x)$ can have at most one negative real root. In this case, we have

$$f(-1) = -2n - 2 < 0, f(0) = 1 > 0$$

So, the negative real root lies between -1 and 0

76 (c)

Given, ${}^{n+1}C_{n-2} - {}^{n+1}C_{n-1} \leq 50$

$$\Rightarrow \frac{(n-1)!}{3!(n-2)!} - \frac{(n+1)!}{2!(n-1)!} \leq 50$$

$$\Rightarrow \frac{(n+1)!}{3!} \left[\frac{1}{(n-2)!} - \frac{3}{(n-1)!} \right] \leq 50$$

$$\Rightarrow (n+1)! \left(\frac{n-1-3}{(n-1)!} \right) \leq 300$$

$$\Rightarrow (n+1)n(n-4) \leq 300$$

For $n = 8$, it satisfies to the above inequality

But $n = 1$ it does not satisfy the above inequality

77 (d)

We have,

$$f(\theta) = \sec^2 \theta + \cos^2 \theta = (\sec \theta - \cos \theta)^2 + 2 \geq 2$$

$$\Rightarrow f(\theta) \in [2, \infty)$$

78 (d)

Let a_1, a_2, \dots, a_n be n positive integers such that

$$a_1 a_2 \dots a_n = n^n$$

Since, $AM \geq GM$

$$\therefore \frac{a_1 + a_2 + \dots + a_n}{n} \geq (a_1 a_2 \dots a_n)^{1/n}$$

$$\Rightarrow \frac{a_1 + a_2 + \dots + a_n}{n} \geq n$$

$$\Rightarrow a_1 + a_2 + \dots + a_n \geq n^2$$

79 (a)

$$\log_2(x^2 - 3x + 18) < 4$$

$$\Rightarrow x^2 - 3x + 18 < 16 \quad (\text{If } \log_a b < c \Rightarrow b < a^c)$$

$$\Rightarrow x^2 - 3x + 2 < 0$$

$$\Rightarrow (x-1)(x-2) < 0$$

$$\Rightarrow x \in (1, 2)$$

80 (d)

We have,

$$[x]^2 = [x+6]$$

$$\Rightarrow [x]^2 = [x] + 6$$

$$\Rightarrow [x]^2 - [x] - 6 = 0$$

$$\Rightarrow ([x]-3)([x]+2) = 0$$

$$\Rightarrow [x] = 3, [x] = -2$$

$$\Rightarrow x \in [3, 4) \text{ or } x \in [-2, -1) \Rightarrow x$$

$$\in [-2, -1) \cup [3, 4)$$

81 (b)

Using $AM \geq GM$

$$\frac{\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b}}{3} \geq \sqrt[3]{\frac{abc}{(a+b)(b+c)(c+a)}} \dots (i)$$

Again, using $AM \geq GM$

$$\frac{a+b}{2} \geq \sqrt{ab}, \frac{b+c}{2} \geq \sqrt{bc}, \frac{c+a}{2} \geq \sqrt{ca}$$

$$\Rightarrow (a+b)(b+c)(c+a) \geq 8abc$$

$$\Rightarrow \sqrt[3]{\frac{abc}{(a+b)(b+c)(c+a)}} \leq \frac{1}{2}$$

\therefore From Eq. (i)

$$\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b} \geq \frac{3}{2}$$

82 (a)

Two curves $y = e^{|x|}$ and $y = |x|$ does not intersect. So, the equation $e^{|x|} - |x| = 0$ has no solution

83 (b)

$$|p-q| = \begin{cases} p-q, p \geq q \\ q-p, p < q \end{cases}$$

$$\therefore \min(p, q) = \begin{cases} p, p < q \\ q, q < p \end{cases}$$

$$\Rightarrow \text{RHS} = \frac{1}{2}(p+q - |p-q|), \text{ if } p > q$$

$$\Rightarrow \frac{1}{2}(p+q - p+q) = q$$

and LHS $\min(p, q) = q$

$$\therefore \min(p, q) = \frac{1}{2}(p+q - |p-q|)$$

84 (a)

The given equation is

$$1 + |e^x - 1| = e^x(e^x - 2)$$

$$\Rightarrow |e^x - 1| + 2 = (e^x - 1)^2$$

$$\Rightarrow |e^x - 1|^2 - (e^x - 1) - 2 = 0$$

$$\Rightarrow (|e^x - 1| - 2)(|e^x - 1| + 1) = 0$$

$$\Rightarrow |e^x - 1| - 2 = 0 \quad [\because |e^x - 1| + 1 \neq 0]$$

$$\Rightarrow e^x - 1 = \pm 2 \Rightarrow e^x = 3, -1$$

$$\Rightarrow e^x = 3 \Rightarrow x = \log_e 3 \quad [\because e^x > 0 \text{ for all } x]$$

85 (c)

We have,

$$\log_{\sin \frac{2\pi}{3}}(x^2 - 3x + 2) \geq 2$$

$$\Rightarrow (x^2 - 3x + 2) \leq \left(\frac{\sqrt{3}}{2}\right)^2 \text{ and } x^2 - 3x + 2 > 0$$

$$\Rightarrow 4x^2 - 12x + 5 \leq 0 \text{ and } x^2 - 3x + 2 > 0$$

$$\Rightarrow (2x-1)(2x-5) \leq 0 \text{ and } (x-1)(x-2) > 0$$

$$\Rightarrow \frac{1}{2} \leq x \leq \frac{5}{2} \text{ and } x < 1 \text{ or } x > 2$$

$$\Rightarrow x \in [1/2, 1) \cup (2, 5/2]$$

86 (c)

We have, $x^2 + 6x - 27 > 0$

$$\Rightarrow (x + 9)(x - 3) > 0 \Rightarrow x < -9 \text{ or } x > 3$$

$$\Rightarrow x \in (-\infty, -9) \cup (3, \infty) \dots(i)$$

And $x^2 - 3x - 4 < 0$

$$\Rightarrow (x - 4)(x + 1) < 0$$

$$\Rightarrow -1 < x < 4 \dots(ii)$$

From relations (i) and (ii), we get

$$3 < x < 4$$

87 (d)

We have,

$$\frac{1}{2}\{(x + y) + |x - y|\} = x$$

$$\Rightarrow \frac{1}{2}\{(x + y) + |x - y|\} = \frac{1}{2}\{(x + y) + (x - y)\}$$

$$\Rightarrow |x - y| = x - y \Rightarrow x \geq y$$

88 (c)

Given, $\frac{4x-1}{3x+1} - 1 \geq 0$

$$\Rightarrow \frac{x-2}{3x+1} \geq 0$$

$$\Rightarrow x - 2 \geq 0 \text{ and } 3x + 1 > 0$$

$$\text{Or } x - 2 \leq 0 \text{ and } 3x + 1 < 0$$

$$\Rightarrow x \geq 2 \text{ and } x < -\frac{1}{3}$$

$$\text{Or } x \leq 2 \text{ and } x > -\frac{1}{3}$$

$$\Rightarrow x \in \left(-\infty, -\frac{1}{3}\right) \cup [2, \infty)$$

89 (c)

Given inequality holds only, if

$$\sin^2 \alpha_i = 1 \text{ or } \alpha_i = \pm \frac{\pi}{2}, \frac{3\pi}{2}; (i = 2, 3, \dots, n)$$

\Rightarrow Number of solutions = $3 \times 3 \times 3 \times \dots \times (n - 1)$ times

$$= 3^{n-1}$$

90 (b)

We have, $e^x = x(x + 1), x < 0$

Consider the curves $y = e^x$ and $y = x(x + 1)$ for $x < 0$. Graphs of these two curve intersect at exactly one point. So, the equation $e^x = x(x + 1)$ has exactly one real root

91 (b)

Draw graphs of $y = 1 - x + [x]$ and $y = \frac{1}{x} - \frac{1}{[x]}$

These two curves intersect it infinitely many points

92 (d)

We have, $2^x + 2^{|x|} \geq 2\sqrt{2}$

Following cases arise:

CASE I When $x \geq 0$

In this case, we have

$$2^x + 2^x \geq 2\sqrt{2} \Rightarrow 2^x \geq 2^{1/2} \Rightarrow x \geq \frac{1}{2} \Rightarrow x$$

$$\in \left[\frac{1}{2}, \infty\right)$$

CASE II When $x < 0$

In this case, we have

$$2^x + 2^{-x} \geq 2\sqrt{2}$$

$$\Rightarrow (2^x)^2 - 2\sqrt{2} \times 2^x + 1 \geq 0$$

$$\Rightarrow (2^x - \sqrt{2})^2 - 1 \geq 0$$

$$\Rightarrow (2^x - \sqrt{2} - 1)(2^x - \sqrt{2} + 1) \geq 0$$

$$\Rightarrow 2^x \leq \sqrt{2} - 1 \text{ or } 2^x \geq \sqrt{2} + 1$$

$$\Rightarrow x \leq \log_2(\sqrt{2} - 1) \text{ or } x \geq \log_2(\sqrt{2} + 1)$$

$$\Rightarrow x \leq \log_2(\sqrt{2} - 1) \Rightarrow x \in (-\infty, \log_2(\sqrt{2} - 1))$$

$$\text{Hence, } x \in (-\infty, \log_2(\sqrt{2} - 1)) \cup [1/2, \infty)$$

93 (c)

$$\sin^4 \frac{x}{3} + \cos^4 \frac{x}{3} > \frac{1}{2} \Rightarrow 1 - \frac{1}{2} \sin^2 \frac{2x}{3} > \frac{1}{2}$$

$$\Rightarrow \sin^2 \frac{2x}{3} < 1 \Rightarrow \frac{2x}{3} \in \left(R - (2n + 1)\frac{\pi}{2}\right)$$

$$\Rightarrow x \in R - \left(\frac{3n\pi}{2} + \frac{3\pi}{4}\right); n \in I$$

94 (a)

We have,

$$(\log_5 x)^2 + (\log_5 x) < 2$$

$$\Rightarrow (\log_5 x)^2 + (\log_5 x) - 2 < 0$$

$$\Rightarrow (\log_5 x + 2)(\log_5 x - 1) < 0$$

$$\Rightarrow -2 < \log_5 x < 1 \Rightarrow 5^{-2} < x < 5 \Rightarrow x \in \left(\frac{1}{25}, 5\right)$$

96 (d)

$$\because \sin x + 2\sqrt{2} \cos x \geq (\sqrt{3})^2$$

$$\Rightarrow \sin x + 2\sqrt{2} \cos x \geq 3$$

$$\Rightarrow \sin \left(a + \cos^{-1} \frac{1}{3}\right) \geq 1$$

$$\Rightarrow \sin \left(x + \cos^{-1} \frac{1}{3}\right) = 1 \quad (\because \sin x \text{ cannot be greater than } 1)$$

$$\therefore x = n\pi + (-1)^n \frac{\pi}{2} - \cos^{-1} \frac{1}{3}$$

For solution in the interval $[-2\pi, 2\pi]$, $n = 0, 1, -1, -2$

97 (c)

$$x^{(\log_{10} x)^2 - 3 \log_{10} x + 1} > 1000 = 10^3$$

$$\Rightarrow [(\log_{10} x)^2 - 3 \log_{10} x + 1] \log_{10} x > 3 \log_{10} 10 = 3$$

$$\Rightarrow (\log_{10} x)^3 - 3(\log_{10} x)^2 + \log_{10} x > 3$$

$$\Rightarrow (\log_{10} x)(\log_{10} x - 3) + 1(\log_{10} x - 3) > 0$$

$$\Rightarrow (\log_{10} x - 3)(\log_{10} x + 1) > 0$$

$$\Rightarrow \log_{10} x - 3 > 0 \Rightarrow \log_{10} x > 3$$

$$\Rightarrow x > 10^3 = 1000$$

$$\Rightarrow x \in (1000, \infty)$$

98 (d)

$x^{12} - x^9 + x^4 - x + 1 > 0$, three cases arise

Case I When $x \leq 0$

$$x^{12} > 0, -x^9 > 0, x^4 > 0, -x > 0$$

$$\Rightarrow x^{12} - x^9 + x^4 - x + 1 > 0, \forall x \leq 0 \dots(i)$$

Case II When $0 < x \leq 1$

$$x^9 < x^4, x < 1 \Rightarrow -x^9 + x^4 > 0 \text{ and } 1 - x > 0$$

$$\therefore x^{12} - x^9 + x^4 - x + 1 > 0, \forall 0 < x \leq 1 \dots(ii)$$

Case III When $x > 1$

$$x^{12} > x^9, x^4 > x$$

$$\Rightarrow x^{12} - x^9 + x^4 - x + 1 > 0, \forall x > 1 \dots(iii)$$

\therefore From Eqs. (i), (ii) and (iii) the above equation hold for $x \in R$

100 (a)

We have,

$$\sqrt{x+1} - \sqrt{x-1} = \sqrt{4x-1}$$

$$\Rightarrow x+1+x-1-2\sqrt{x^2-1} = 4x-1$$

$$\Rightarrow -2\sqrt{x^2-1} = 2x-1$$

$$\Rightarrow 4(x^2-1) = 4x^2-4x+1 \Rightarrow 4x-5=0 \Rightarrow x = \frac{5}{4}$$

This value of x does not satisfy the given equation.

So, the equation has no solution

101 (d)

The LHS of the given inequality is meaningful for $x > 0$ and $x \neq 1$

Now,

$$\log_3 x - \log_x 27 < 2$$

$$\Rightarrow \log_3 x - 3 \log_x 3 < 2$$

$$\Rightarrow \log_3 x - \frac{3}{\log_3 x} < 2$$

$$\Rightarrow \frac{(\log_3 x)^2 - 3 - 2(\log_3 x)}{\log_3 x} < 0$$

$$\Rightarrow \frac{(\log_3 x - 3)(\log_3 x + 1)}{(\log_3 x - 0)} < 0$$

$$\Rightarrow \log_3 x < -1 \text{ or } 0 < \log_3 x < 3$$

$$\Rightarrow x < 3^{-1} \text{ or } 3^0 < x < 3^3 \Rightarrow x < \frac{1}{3} \text{ or } 1 < x < 27$$

Also, $x > 0$ and $x \neq 1$

$$\therefore x \in (0, 1/3) \cup (1, 27)$$

102 (c)

Given inequation is $x^2 - 2x + 5 \leq 0$

\therefore Roots are

$$x = \frac{-2 \pm \sqrt{4 - 20}}{2} = \frac{2 \pm 4i}{2}$$

\therefore Roots are imaginary, therefore no real solutions exist

103 (b)

We have,

$$\frac{2^{\sin^2 x} + 2^{\cos^2 x}}{2} \geq \sqrt{2^{\sin^2 x} \times 2^{\cos^2 x}} \quad [\text{Using A. M.} \geq \text{G. M.}]$$

$$\Rightarrow 2^{\sin^2 x} + 2^{\cos^2 x} \geq 2\sqrt{2} \Rightarrow 2^{\sin^2 x} + 2^{\cos^2 x} \geq 2$$

104 (a)

Given that, $a^2 + b^2 + c^2 = 1 \dots(i)$

Now, $(a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab+bc+ca) \geq 0$

$$\Rightarrow 2(ab+bc+ca) \geq -1 \quad [\text{from Eq.(i)}]$$

$$\Rightarrow ab+bc+ca \geq -\frac{1}{2} \dots(ii)$$

Also, $a^2 + b^2 + c^2 - ab - bc - ca$

$$= \frac{1}{2} \{(a-b)^2 + (b-c)^2 + (c-a)^2\} \geq 0$$

$$\Rightarrow ab+bc+ca \leq a^2 + b^2 + c^2$$

$$\Rightarrow ab+bc+ca \leq 1 \quad [\text{from Eq.(i)}] \dots(iii)$$

From relation (ii) and (iii), we get

$$-\frac{1}{2} \leq ab+bc+ca \leq 1$$

105 (b)

We have,

$$\sqrt{4x+9} - \sqrt{11x+1} = \sqrt{7x+4}$$

$$\Rightarrow \sqrt{4x+9} - \sqrt{7x+4} = \sqrt{11x+1}$$

$$\Rightarrow 4x + 9 + 7x + 4 - 2\sqrt{(4x + 9)(7x + 4)}$$

$$= 11x + 1$$

$$\Rightarrow -2\sqrt{(4x + 9)(7x + 4)} = -12$$

$$\Rightarrow (4x + 9)(7x + 4) = 36$$

$$\Rightarrow 28x^2 + 79x = 0 \Rightarrow x = 0, -\frac{79}{28}$$

Clearly, only $x = 0$ satisfies the given equation

106 (a)

$$\text{Since, } -3 < x + \frac{2}{x} < 3$$

$$\Rightarrow -3 < \frac{(x^2 + 2)x}{x^2} < 3$$

$$\Rightarrow -3x^2 < (x^2 + 2)x < 3x^2 \quad (x \neq 0)$$

$$\Rightarrow x(x^2 + 3x + 2) > 0$$

$$\text{And } x(x^2 - 3x + 2) < 0 \quad (x \neq 0)$$

$$\Rightarrow x(x + 1)(x + 2) > 0$$

$$\text{And } x(x - 1)(x - 2) < 0$$

$$\Rightarrow x \in (-2, -1) \cup (0, \infty) \dots(i)$$

$$\text{And } x \in (-\infty, 0) \cup (1, 2) \dots(ii)$$

From relations (i) and (ii), we get

$$x \in (-2, -1) \cup (1, 2)$$

107 (b)

We have a, b, c are sides of a triangle

$$\therefore b + c - a > 0, c + a - b > 0, a + b - c > 0$$

$$\text{Let } x = b + c - a, y = c + a - b, z = a + b - c$$

$$\Rightarrow y + z = 2a, z + x = 2b, x + y = 2c$$

$$\text{Now, } \frac{a}{b+c-a} + \frac{b}{c+a-b} + \frac{c}{a+b-c}$$

$$= \frac{y+z}{2x} + \frac{z+x}{2y} + \frac{x+y}{2z}$$

$$= \frac{1}{2} \left(\frac{x}{y} + \frac{y}{z} + \frac{z}{x} + \frac{x}{z} + \frac{y}{x} + \frac{z}{y} \right)$$

$$\geq \frac{6}{2} \left(\frac{y}{x} \cdot \frac{x}{y} \cdot \frac{y}{z} \cdot \frac{z}{y} \cdot \frac{z}{x} \cdot \frac{x}{z} \right) \quad (\because \text{AM} \geq \text{GM})$$

$$= 3$$

108 (b)

Let d_1, d_2 be the lengths of diagonals and θ be the angle between them. Then,

$$\text{Area} = \frac{1}{2} d_1 d_2 \sin \theta \Rightarrow a^2$$

$$= \frac{1}{2} d_1 d_2 \sin \theta \Rightarrow d_1 d_2 = \frac{2a^2}{\sin \theta}$$

Using A. M. \geq G. M., we have

$$\frac{d_1 + d_2}{2} \geq \sqrt{d_1 d_2} \Rightarrow d_1 + d_2 \geq 2 \sqrt{\frac{2a^2}{\sin \theta}} \geq 2\sqrt{2}a$$

109 (c)

$$\text{We have, } |2x - 3| < |x + 5|$$

$$\Rightarrow |2x - 3| - |x + 5| < 0$$

$$\Rightarrow \begin{cases} 3 - 2x + x + 5 < 0, x \leq -5 \\ 3 - 2x - x - 5 < 0, -5 < x \leq \frac{3}{2} \\ 2x - 3 - x - 5 < 0, x > \frac{3}{2} \end{cases}$$

$$\Rightarrow \begin{cases} x > 8, x \leq -5 \\ x > -\frac{2}{3}, -5 < x \leq \frac{3}{2} \\ x < 8, x > \frac{3}{2} \end{cases}$$

$$\Rightarrow x \in \left(-\frac{2}{3}, \frac{3}{2}\right] \cup \left(\frac{3}{2}, 8\right)$$

$$\Rightarrow x \in \left(-\frac{2}{3}, 8\right)$$

110 (c)

We have,

$$|a_n| < 2 \text{ i.e. } -2 < a_n < 2$$

$$\therefore \max(1 + a_1x + a_2x^2 + \dots + a_nx^n)$$

$$= 1 + 2|x| + 2|x|^2 + \dots + 2|x|^n$$

$$= 1 + 2|x| \left\{ \frac{1 - |x|^n}{1 - |x|} \right\}$$

$$= 1 + 2 \cdot \frac{1}{3} \left\{ \frac{1 - 1/3^n}{1 - 1/3} \right\} > 0$$

and,

$$\min(1 + a_1x + a_2x^2 + \dots + a_nx^n)$$

$$= 1 - 2|x| - 2|x|^2 - \dots - 2|x|^n$$

$$= -2[1 + |x| + |x|^2 + \dots + |x|^n] + 3$$

$$= -2 \left\{ \frac{1 - |x|^n}{1 - |x|} \right\} + 3$$

$$= -2 \left\{ \frac{1 - 1/3^n}{1 - 1/3} \right\} + 3 > 0$$

Thus, the curve $y = 1 + a_1x + a_2x^2 + \dots + a_nx^n$

does not meet x -axis for $|x| < 1/3$ and $|a_n| < 2$

Hence, the equation has no real roots

112 (c)

$$\text{Since, } |r| < 1 \Rightarrow -1 < r < 1$$

$$\text{Also, } a = 5(1 - r)$$

$$\Rightarrow 0 < a < 10 \quad \left[\begin{array}{l} \because \text{ at } r = -1, \quad a = 0 \\ \text{and at } r = 1, \quad a = 0 \end{array} \right]$$

113 (a)

Consider the curves $y = e^{x-8}$ and $y = 17 - 2x$.
These two curves intersect at (8, 1) only. Hence,
the equation $e^{x-8} + 2x - 17 = 0$ has exactly one
root which is equal to 8

114 (b)

Let $x^2 + 18x + 30 = y$. Then,
 $x^2 + 18x + 30 = 2\sqrt{x^2 + 18x + 45}$
 $\Rightarrow y = 2\sqrt{y + 15}$
 $\Rightarrow y^2 - 4y - 60 = 0 \Rightarrow (y - 10)(y + 6) = 0 \Rightarrow y = 10$
 $\therefore x^2 + 18x + 30 = y \Rightarrow x^2 + 18x + 20 = 0$
 \therefore Product of roots = 20

115 (a)

Since, $2 \leq |x - 3| < 4$
 $\Rightarrow 2 \leq x - 3 < 4$
Or $2 \leq -(x - 3) < 4$
 $\Rightarrow 5 \leq x < 7$ or $-1 \leq -x < 1$
 $\Rightarrow 5 \leq x < 7$ or $-1 < x \leq 1$
 $\therefore x \in (-1, 1] \cup [5, 7)$

116 (b)

Given that, $x = \frac{a+2b}{a+b}$ and $y = \frac{a}{b}$
 $\therefore x = \frac{a+2b}{a+b} = \frac{\frac{a}{b} + 2}{1 + \frac{a}{b}} = 1 + \frac{1}{\frac{a}{b} + 1}$
 $\Rightarrow x = 1 + \frac{1}{y+1} \quad [\because y = \frac{a}{b} \text{ and } y^2 > 2 \text{ (given)}]$
Which shows $x^2 < 2 \quad [\because \frac{1}{y+1} < \text{as } y > 1]$

117 (c)

Using A.M. \geq G.M., we have
 $5^{\sin x - 1} + 5^{-\sin x - 1} \geq 2\sqrt{5^{\sin x - 1} \times 5^{-\sin x - 1}}$
 $\Rightarrow 5^{\sin x - 1} + 5^{-\sin x - 1} \geq \frac{2}{5}$

118 (d)

As we know, if $ax + bx + c > 0$, then $a > 0$ and $D < 0$
 $\therefore (2)^2 - 4(n - 10) < 0 \Rightarrow n > 11$

119 (a)

Since, AM \geq GM
 $\Rightarrow \frac{bcx + cay + abz}{3} \geq (a^2 b^2 c^2 \cdot xyz)^{1/3}$
 $\Rightarrow bcx + cay + abz \geq 3abc \quad (\because xyz = abc)$

120 (b)

$$\text{Since, } \frac{(1+a_1)}{2} \geq \sqrt{1 \cdot a_1} = \sqrt{a_1}$$

$$\frac{(1+a_2)}{2} \geq \sqrt{1 \cdot a_2} = \sqrt{a_2}$$

\vdots

$$\frac{(1+a_n)}{2} \geq \sqrt{1 \cdot a_n} = \sqrt{a_n}$$

$$\Rightarrow \frac{1}{2^n} (1+a_1)(1+a_2) \dots (1+a_n) \geq \sqrt{a_1 a_2 \dots a_n} = 1$$

$$\Rightarrow (1+a_1)(1+a_2) \dots (1+a_n) \geq 2^n$$

121 (a)

Using A.M. \geq G.M., we have
 $a + b \geq 2\sqrt{ab}$, $b + c \geq 2\sqrt{bc}$ and $c + a \geq 2\sqrt{ca}$
 $\Rightarrow (a+b)(b+c)(c+a) \geq 8abc$
 $\Rightarrow (2-a)(2-b)(2-c) \geq 8abc \quad \left[\begin{array}{l} \because a+b+c=2 \\ \therefore b+c=2-a \text{ etc} \end{array} \right]$

122 (a)

We have, $x^2 + y^2 + z^2 = 27$
Now,
 $\frac{(x^2)^{3/2} + (y^2)^{3/2} + (z^2)^{3/2}}{3} \geq \left(\frac{x^2 + y^2 + z^2}{3} \right)^{3/2}$
 $\Rightarrow x^3 + y^3 + z^3 \geq 81$

123 (c)

Given, $2x - 7 < 11$, $3x + 4 < -5$
 $\Rightarrow x < 9$, $x < -3$
 $\Rightarrow x < -3$
 $\therefore x$ lies in the interval $(-\infty, -3)$

124 (c)

Let
 $f(x) = x^8 - x^5 - \frac{1}{x} + \frac{1}{x^4} = \frac{x^{12} - x^9 - x^3 + 1}{x^4}$
 $= \frac{(x^9 - 1)(x^3 - 1)}{x^4}$

Clearly, $f(x) \geq 0$ for all $x < 0$ and it is not defined for $x = 0$

For $0 < x < 1$, we have

$$x^9 - 1 < 0 \text{ and } x^3 - 1 < 0 \Rightarrow f(x) > 0$$

For $x \geq 1$, we have $x^9 - 1 \geq 0$ and $x^3 - 1 \geq 0 \Rightarrow f(x) \geq 0$

Hence, $f(x) \geq 0$ for all $x \neq 0$

125 (c)

We have,
 $5^{(1/4)(\log_5 x)^2} \geq 5 x^{(1/5)(\log_5 x)}$
 $\frac{1}{4}(\log_5 x)^2 \log_5 5 \geq \log_5 5 + \frac{1}{5}(\log_5 x) \log_5 x$

$$\begin{aligned} &\Rightarrow (\log_5 x)^2 \geq 20 \\ &\Rightarrow (\log_5 x)^2 - (2\sqrt{5})^2 \geq 0 \\ &\Rightarrow \log_5 x \leq -2\sqrt{5} \text{ or } \log_5 x \geq 2\sqrt{5} \\ &\Rightarrow x \leq 5^{-2\sqrt{5}} \text{ or } x \geq 5^{2\sqrt{5}} \\ &\Rightarrow x \\ &\in (0, 5^{-2\sqrt{5}}] \\ &\cup [5^{2\sqrt{5}}, \infty) \quad [\log_5 x \text{ is defined for } x > 0] \end{aligned}$$

126 (a)

We know that,

$$AM \geq GM$$

$$\Rightarrow \frac{a+b}{2} \geq \sqrt{ab}$$

$$\Rightarrow 4 \geq \sqrt{ab} \quad (\because a+b=8 \text{ given})$$

$$\Rightarrow ab \leq 16$$

Equality holds when number are equal. So, ab is equal to 16 when $a=4, b=4$

127 (a)

Curves $y = \cos x$ and $y = -|x|$ do not intersect. So, the equation $\cos x + |x| = 0$ has no real root

128 (d)

Using A. M. \geq G. M., we have

$$\frac{\cos^3 x}{\sin x} + \frac{\sin^3 x}{\cos x} \geq 2 \sqrt{\frac{\cos^3 x}{\sin x} \times \frac{\sin^3 x}{\cos x}} \text{ for all } x \in (0, \pi/2)$$

$$\Rightarrow \frac{\cos^3 x}{\sin x} + \frac{\sin^3 x}{\cos x} \geq \sin 2x \text{ for all } x \in (0, \pi/2)$$

$$\Rightarrow \frac{\cos^3 x}{\sin x} + \frac{\sin^3 x}{\cos x} \geq 1 \text{ for all } x \in (0, \pi/2)$$

129 (d)

$$x^2 + 4ax + 20$$

$$\therefore (4a)^2 - 4 \times 2 < 0$$

[\because if $f(x) > 0$, then]

$$\Rightarrow 16a^2 < 8 \Rightarrow a^2 < \frac{1}{2}$$

$$\Rightarrow -\frac{1}{\sqrt{2}} < a < \frac{1}{\sqrt{2}}$$

130 (b)

Using A. M. \geq G. M., we have

$$\frac{a+b}{2} > \sqrt{ab} \quad [\because a \neq b]$$

$$\Rightarrow a+b > 2\sqrt{ab}$$

131 (a)

We have,

$$x^2 \cdot 2^{x+1} + 2^{|x-3|+2} = x^2 \cdot 2^{|x-3|+4} + 2^{x-1}$$

Now, two cases arise

CASE I When $x \geq 3$:

In this case, we have $|x-3| = x-3$

So, the given equation reduces to

$$x^2 \cdot 2^{x+1} + 2^{x-1} = x^2 \cdot 2^{x+1} + 2^{x-1}$$

Which is an identity in x and hence it is true for all $x \geq 3$

CASE II When $x < 3$:

In this case, we have $|x-3| = -(x-3)$

So, the given equation reduces to

$$x^2 \cdot 2^{x+1} + 2^{-x+5} = x^2 \cdot 2^{-x+7} + 2^{x-1}$$

$$\Rightarrow x^2 2^{x+1} - 2^{x-1} = x^2 \cdot 2^{-x+7} - 2^{-x+5}$$

$$\Rightarrow 2^{x-1}(4x^2 - 1) = 2^{-x+5}(4x^2 - 1)$$

$$\Rightarrow 2^{2x}(4x^2 - 1) = 2^6(4x^2 - 1)$$

$$\Rightarrow (2^{2x} - 2^6)(4x^2 - 1) = 0$$

$$\Rightarrow 2x = 6 \text{ or } 4x^2 - 1 = 0$$

$$\Rightarrow x = 3 \text{ or } x = \pm \frac{1}{2}$$

$$\text{But, } x < 3. \text{ Therefore, } x = \pm \frac{1}{2}$$

Hence, the given equation has no negative integral root

132 (b)

We have,

$$2^{\sin^2 x} \cdot 3^{\cos^2 y} \cdot 4^{\sin^2 z} \cdot 5^{\cos^2 \omega} \geq 120$$

$$\Rightarrow 2^{\sin^2 x} \cdot 3^{\cos^2 y} \cdot 4^{\sin^2 z} \cdot 5^{\cos^2 \omega} \geq 2 \times 3 \times 4 \times 5$$

$$\Rightarrow \sin^2 x \log 2 + \cos^2 y \log 3 + \sin^2 z \log 4$$

$$+ \cos^2 \omega \log 5$$

$$\geq \log 2 + \log 3 + \log 4 + \log 5$$

$$\Rightarrow \cos^2 x \log 2 + \sin^2 y \log 3$$

$$+ \cos^2 z \log 4 + \sin^2 \omega \log 5 \leq 0$$

$$\Rightarrow \cos^2 x = 0, \sin^2 y$$

$$= 0, \cos^2 z = 0 \text{ and } \sin^2 \omega = 0$$

$$\Rightarrow x = m\pi \pm \frac{\pi}{2}, m \in \mathbb{Z}; y = n\pi, n \in \mathbb{Z}$$

$$z = r\pi \pm \frac{\pi}{2}, r \in \mathbb{Z}; \omega = t\pi, t \in \mathbb{Z}$$

But, $x, y, z, \omega \in [0, 10]$

$$\therefore x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, y = 0, \pi, 2\pi, 3\pi, z = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$$

and $\omega = 0, \pi, 2\pi, 3\pi$

Hence, the number of ordered 4-tuples is $3 \times 4 \times 3 \times 4 = 144$

133 (d)

We have,

$$\log_b a + \log_a b + \log_d c + \log_c d$$

$$= \left(\log_b a + \frac{1}{\log_b a} \right) + \left(\log_d c + \frac{1}{\log_d c} \right) \geq 2 + 2 = 4$$

134 (c)

We have,

$$\log_{1/3}(2^{x+2} - 4^x) \geq -2$$

$$\Rightarrow 2^{x+2} - 4^x \leq \left(\frac{1}{3}\right)^{-2} \text{ and } 2^{x+2} - 4^x > 0$$

$$\Rightarrow 4(2^x) - (2^x)^2 \leq 9 \text{ and } 2^x(2^2 - 2^x) > 0$$

$$\Rightarrow (2^x)^2 - 4(2^x) + 9 \geq 0 \text{ and } 2^x < 2^2$$

$$\Rightarrow x < 2 \text{ [}\because (2^x)^2 - 4(2^x) + 9 > 0 \text{ for all } x \in \mathbb{R}]$$

$$\Rightarrow x \in (-\infty, 2)$$

135 (c)

Given, $\frac{2x}{(2x+1)(x+2)} - \frac{1}{(x+1)} > 0$

$$\Rightarrow \frac{-3x-2}{(x+1)(x+2)(2x+1)} > 0$$

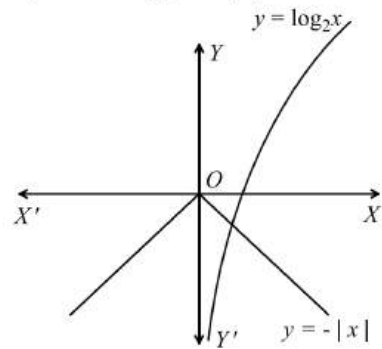
Equating each factor equal to 0, we have

$$x = -2, -1, -\frac{2}{3}, -\frac{1}{2}$$

It is clear $-\frac{2}{3} < x < -\frac{1}{2}$ or $-2 < x < -1$

136 (b)

We observe that the curves $y = \log_2 x$ and $y = -|x|$ intersect at exactly one point. So, the equation $\log_2 x + |x| = 0$ has exactly one real root



137 (a)

Using A. M. \geq G. M., we have

$$\frac{bcx + cay + abz}{3} \geq (bcx \times cay \times abz)^{1/3}$$

$$\Rightarrow bcx + cay + abz \geq 3(a^2b^2c^2 \times xyz)^{1/3}$$

$$\Rightarrow bcx + cay + abz \geq 3abc \quad [\because xyz = abc]$$

138 (a)

We have,

$$\sqrt{3x^2 + 6x + 7} + \sqrt{5x^2 + 10x + 14}$$

$$\leq 4 - 2x - x^2$$

$$\Rightarrow \sqrt{3(x+1)^2 + 4} + \sqrt{5(x+1)^2 + 9}$$

$$\leq (x+1)^2 + 5$$

Clearly, LHS ≥ 5 and LHS ≤ 5

So, the inequation holds when each side is equal to 5

This is true when $x = -1$

Hence, the given inequation has exactly one solution

139 (b)

Let $a_1, a_2, a_3, \dots, a_n$ be the lengths of n parts of the stick. Then,

$$a_1 + a_2 + a_3 + \dots + a_n = 20 \text{ and } a_1 a_2 a_3 \dots a_n > 1$$

Now, A. M. \geq G. M.

$$\Rightarrow \frac{a_1 + a_2 + \dots + a_n}{n} \geq (a_1 a_2 \dots a_n)^{1/n}$$

$$\Rightarrow \frac{20}{n} > 1 \Rightarrow n < 20$$

\therefore Maximum possible value of n is 19

140 (b)

\because AM $>$ GM

$$\frac{(a+b-c) + (b+c-a)}{2} > \sqrt{(a+b-c)(b+c-a)}$$

$$\Rightarrow b > \sqrt{(a+b-c)(b+c-a)} \dots(i)$$

Similarly, $\frac{(b+c-a) + (c+a-b)}{2} >$

$$\sqrt{(b+c-a)(c+a-b)}$$

$$\Rightarrow c > \sqrt{(b+c-a)(c+a-b)} \dots(ii)$$

and $\frac{(c+a-b) + (a+b-c)}{2} > \sqrt{(c+a-b)(a+b-c)}$

$$\Rightarrow a > \sqrt{(c+a-b)(a+b-c)} \dots(iii)$$

On multiplying relations (i), (ii) and (iii), we get

$$abc > (a+b-c)(b+c-a)(c+a-b)$$

$$\Rightarrow (a+b-c)(b+c-a)(c+a-b) - abc < 0$$

141 (c)

We have,

$$\log_{16} x^3 + (\log_2 \sqrt{x})^2 < 1$$

$$\Rightarrow \frac{3}{4} \log_2 x + \frac{1}{4} (\log_2 x)^2 < 1$$

$$\Rightarrow (\log_2 x)^2 + 3 \log_2 x - 4 < 0$$

$$\Rightarrow (\log_2 x + 4)(\log_2 x - 1) < 0$$

$$\Rightarrow -4 < \log_2 x < 1 \Rightarrow 2^{-4} < x < 2 \Rightarrow x \in (1/16, 2)$$

Also, LHS of the given inequality is defined for $x > 0$

Hence, $x \in (1/16, 2)$

142 (b)

Since, $\sin x \leq \cos^2 x$, because $\cos x$ must be a positive proper fraction

$$\sin^2 x + \sin x - 1 \leq 0$$

$$\text{Or } \left(\sin x + \frac{1}{2}\right)^2 - \frac{5}{4} \leq 0$$

From the definition of logarithm

$$\sin x > 0, \cos x > 0, \cos x \neq 1$$

$$\therefore \sin x + \frac{1}{2} \leq \frac{\sqrt{5}}{2},$$

$$\Rightarrow 0 < \sin x \leq \frac{\sqrt{5}-1}{2}$$

143 (d)

$$\text{If } f(x) = x^2 + 2bx + 2c^2 \text{ and } g(x) = -x^2 - 2cx + b^2$$

Then, $f(x)$ is minimum and $g(x)$ is maximum at

$$f(x) = \frac{-D}{4a}, \left(\because x = -\frac{-b}{a} \text{ and } f(x) = \frac{-D}{4a} \right)$$

$$\therefore \min\{f(x)\} = \frac{-(4b^2 - 8c^2)}{4} = (2c^2 - b^2)$$

$$\text{And } \max\{g(x)\} = -\frac{(4c^2 + 4b^2)}{4(-1)} = (b^2 + c^2)$$

$$\text{Since, } \min f(x) > \max g(x) \Rightarrow 2c^2 - b^2 > b^2 + c^2$$

$$\Rightarrow c^2 > 2b^2 \Rightarrow |c| > \sqrt{2}|b|$$

144 (a)

$$\text{We have, } [x] + (x) = 5$$

$$\text{If } x \leq 2, \text{ then } [x] + (x) \leq 2 + 2 < 5$$

$$\text{If } x \geq 3, \text{ then } [x] + (x) \geq 3 + 3 > 5$$

$$\text{If } 2 < x < 3, \text{ then } [x] + (x) = 2 + 3 = 5$$

Hence, the solution set is $(2, 3)$

145 (a)

We have,

$$|x - x^2 - 1| = |2x - 3 - x^2|$$

$$\Rightarrow |x^2 - x + 1| = |x^2 - 2x + 3|$$

$$\Rightarrow x^2 - x + 1$$

$$= x^2 - 2x + 3 \left[\begin{array}{l} \because x^2 - x + 1 > 0 \\ \text{and } x^2 - 2x + 3 > 0 \text{ for all } x \end{array} \right]$$

$$\Rightarrow x = 2$$

146 (a)

$$\text{Given, } \frac{|x-1|}{x+2} - 1 < 0$$

$$\text{Case I When } x < 1, |x-1| = 1-x$$

$$\therefore \frac{1-x}{x+2} - 1 < 0 \Rightarrow \frac{-2x-1}{x+2} < 0$$

$$\Rightarrow \frac{2x+1}{x+2} > 0 \Rightarrow x < -2 \text{ or } x > -\frac{1}{2}$$

But $x < 1$

$$\therefore x \in (-\infty, -2) \cup \left(-\frac{1}{2}, 1\right)$$

$$\text{Case II When } x \geq 1, |x-1| = x-1$$

$$\therefore \frac{x-1}{x+2} - 1 < 0 \Rightarrow -\frac{3}{x+2} < 0$$

$$\Rightarrow \frac{3}{x+2} > 0$$

$$\Rightarrow x > -2$$

But $x \geq 1$

$$\therefore x \geq 1, \text{ i.e., } x \in [1, \infty) \dots(\text{iii})$$

\therefore From Eqs. (i) and (ii), we get

$$x \in (-\infty, -2) \cup \left(-\frac{1}{2}, \infty\right)$$

147 (d)

$$\text{Given that, } \frac{x+2}{x^2+1} > \frac{1}{2}$$

$$\Rightarrow x^2 - 2x - 3 < 0$$

$$\Rightarrow (x-3)(x+1) < 0$$

$$\Rightarrow -1 < x < 3$$

The integer value of x are 0, 1, 2

\therefore The number of integral solutions are 3

148 (b)

$$\because \tan\left(x + \frac{\pi}{3}\right) \geq 1 \Rightarrow \frac{\pi}{4} \leq x + \frac{\pi}{3} < \frac{\pi}{2}$$

$$\Rightarrow -\frac{\pi}{12} \leq x < \frac{\pi}{6}$$

$$\Rightarrow n\pi - \frac{\pi}{12} \leq x \leq n\pi + \frac{\pi}{6}$$

149 (c)

We have,

$$\frac{1 + (\log_a x)^2}{1 + \log_a x} > 1$$

$$\Rightarrow \frac{1 + (\log_a x)^2}{1 + \log_a x} - 1 > 0$$

$$\Rightarrow \frac{(\log_a x)(\log_a x - 1)}{(1 + \log_a x)} > 0$$

$$\Rightarrow -1 < \log_a x < 0 \text{ or, } \log_a x > 1$$

$$\Rightarrow a^{-1} > x > a^0 \text{ or, } x < a \quad [\because 0 < a < 1]$$

$$\Rightarrow x \in (1, 1/a) \cup (0, a) \quad [\because a > 0]$$

150 (b)

We have,

$$x = \frac{y+2}{y+1}$$

$$\Rightarrow y = \frac{2-x}{x-1}$$

$$\Rightarrow \left(\frac{2-x}{x-1}\right)^2 > 2 \quad [\because y^2 > 2]$$

$$\Rightarrow (2-x)^2 > 2(x-1)^2 \Rightarrow x^2 < 2$$

151 (a)

$$\text{Let } x = y - \frac{3\pi}{4}. \text{ Then,}$$

$$\sin x = -\left(\frac{\cos y + \sin y}{\sqrt{2}}\right) \text{ and } \cos x$$

$$= -\left(\frac{\cos y - \sin y}{\sqrt{2}}\right)$$

$$\Rightarrow \sin x + \cos x = -\sqrt{2} \cos y \text{ and } \sin x \cos x = \frac{1}{2}(2 \cos^2 y - 1)$$

Now,

$$\begin{aligned} & |\sin x + \cos x + \tan x + \sec x + \operatorname{cosec} x + \cot x| \\ &= |(\sin x + \cos x) + (\tan x + \cot x) \\ &\quad + (\sec x + \operatorname{cosec} x)| \\ &= \left| (\sin x + \cos x) + \frac{1}{\sin x \cos x} + \frac{\sin x + \cos x}{\sin x \cos x} \right| \\ &= \left| (\sin x + \cos x) \left(1 + \frac{1}{\sin x \cos x} \right) + \frac{1}{\sin x \cos x} \right| \\ &= \left| -\sqrt{2} \cos y \left(1 + \frac{2}{2 \cos^2 y - 1} \right) + \frac{2}{2 \cos^2 y - 1} \right| \\ &= \left| -\sqrt{2} \cos y - \frac{2(\sqrt{2} \cos y - 1)}{2 \cos^2 y - 1} \right| \\ &= \left| -\sqrt{2} \cos y - \frac{2}{\sqrt{2} \cos y + 1} \right| \\ &= \left| \sqrt{2} \cos y + \frac{2}{\sqrt{2} \cos y + 1} \right| \\ &= \left| \lambda + \frac{2}{\lambda + 1} \right|, \text{ where } \lambda = \sqrt{2} \cos y \\ &= \left| (\lambda + 1) + \frac{2}{\lambda + 1} - 1 \right| \geq \left| (\lambda + 1) + \frac{2}{(\lambda + 1)} \right| - 1 \\ &\geq 2 \sqrt{(\lambda + 1) \times \frac{2}{(\lambda + 1)}} - 1 = 2\sqrt{2} - 1 \text{ [Using AM} \\ &\quad \geq \text{GM]} \end{aligned}$$

152 (b)

$$\frac{9 \cdot 3^{2x} - 6 \cdot 3^x + 4}{9 \cdot 3^{2x} + 6 \cdot 3^x + 4} = \frac{(3^{x+1})^2 - 2(3^{x+1}) + 4}{(3^{x+1})^2 + 2(3^{x+1}) + 4}$$

$$= \frac{t^2 - 2t + 4}{t^2 + 2t + 4} \text{ (where } t = 3^{x+1}) \dots(i)$$

$$\text{Since, } \frac{1}{3} < \frac{x^2 - 2x + 4}{x^2 + 2x + 4} < 3$$

\therefore From Eq.(i), the given expression lies between $1/3$ and 3

153 (c)

Using A. M. \geq G. M., we have

$$4^x + 4^{1-x} \geq 2\sqrt{4^x \times 4^{1-x}} \Rightarrow 4^x + 4^{1-x} \geq 4$$

154 (b)

We have,

$$3^{-|x|} - 2^{|x|} = 0 \Rightarrow 3^{-|x|} = 2^{|x|} \Rightarrow 6^{|x|} = 1 \Rightarrow x = 0$$

156 (d)

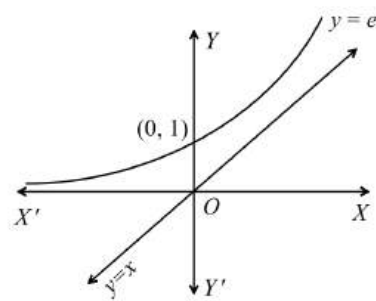
We have,

$$P \leq (x_1 + x_2 + \dots + x_{n-1})(x_2 + x_4 + x_6 + \dots + x_n)$$

$$\Rightarrow P \leq \frac{1}{4}(x_1 + x_2 + \dots + x_n)^2 = \frac{1}{4}$$

157 (b)

We observe that $y = e^{-x}$ and $y = x$ intersect at exactly one point. So, the equation $e^{-x} = x$ has exactly one real root:



158 (b)

We have,

$$3^x + 3^{1-x} - 4 < 0$$

$$\Rightarrow (3^x)^2 - 4(3^x) + 3 < 0$$

$$\Rightarrow (3^x - 1)(3^x - 3) < 0$$

$$\Rightarrow 1 < 3^x < 3 \Rightarrow 0 < x < 1 \Rightarrow x \in (0, 1)$$

160 (c)

We know that $x + \frac{1}{x} \geq 2$ for all $x > 0$

$$\therefore \sin^5 \theta + \operatorname{cosec}^5 \theta \geq 2 \text{ for } 0 < \theta < \pi$$

Hence, the minimum value of $\sin^5 \theta + \operatorname{cosec}^5 \theta$ is 2